Arrangements

Motion Planning and Related Applications

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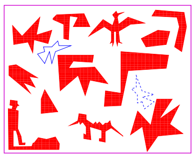
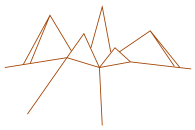
Configuration space

- arrangements $\mathcal{A}(\mathcal{S})$ are used for exact discretization of continuous problems
- a point p in configuration space $\mathcal{C}$ has a property $\Pi(p)$
- if a neighborhood U of p is not intersected by an object in $\mathcal{S}$, the same property $\Pi(q)$ holds for every point $q \in U$
  (the same holds when we restrict the configuration space to an object in $\mathcal{S}$)
- the objects in $\mathcal{S}$ are critical
- the property is invariant in each cell of the arrangement

Invariant, examples



collision-free motion
collides or not



representation of all the different 2D images of a polyhedral terrain
the scene graph

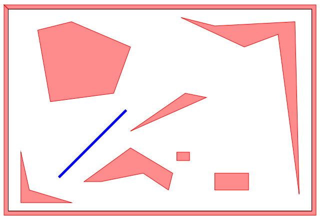
The basic motion planning problem

Let B be a robot system having k degrees of freedom and free to move within a two- or three-dimensional domain V which is bounded by various obstacles whose geometry is known to the system. The motion-planning problem for B is, given the initial and desired final placements of the system B, to determine whether there exists a continuous motion from the initial placement to the final one, during which B avoids collision with the known obstacles, and if so, to plan such a motion.

- its study is strongly tied with the study of arrangements
- many variants: moving obstacles, optimized paths, non-holonomy, uncertainty . . .

Configuration space for translational motion planning

the rod is translating in the room

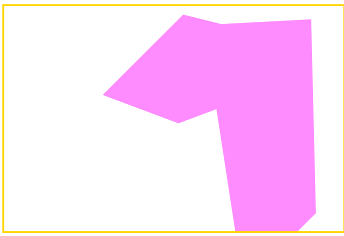


the reference point: the lower end-point of the rod
the configuration space is 2 dimensional

the dimension of the configuration space = the number of degrees of freedom of the moving system

Configuration space obstacles

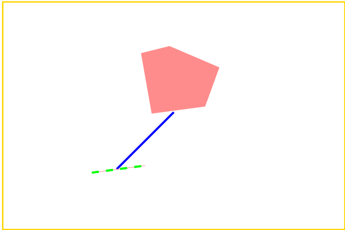
the robot has shrunk to a point
$\Rightarrow$
the obstacles are accordingly expanded



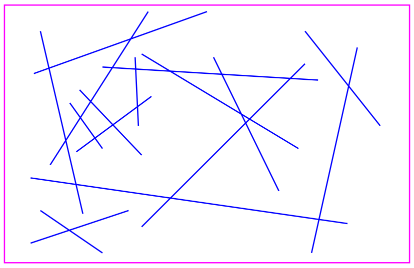
how to efficiently create expanded (C-space) obstacles?

Critical curves in configuration space

the locus of semi-free placements



The arrangement of critical curves in configuration space



suppose we know the initial configuration of the robot

More motion-planning configuration spaces

what is the dimension of the corresponding configuration space, and how do the critical surfaces look?

- a sphere moving among spheres in space
- a polyhedron translating among polyhedra in space
- a polygon translating and rotating among polygons in the plane
- industrial robot arms
- a polyhedron translating and rotating among polyhedra in space

General solutions to the motion-planning problem

- the problem is hard when k (# of dofs) is part of the input [Reif 79], [Hopcroft et al. 84], ...
- the Piano Movers series [Schwartz-Sharir 83], cell decomposition: a doubly-exponential solution, $O((nd)^{3^k})$
  assuming the robot complexity is constant, n is the complexity of the obstacles and d is the algebraic complexity of the problem
- roadmap [Canny 87]: a singly-exponential solution, $n^k(\log n)d^{O(k^2)}$ expected time
  [*Algorithms in Real Algebraic Geometry*, Basu-Pollack-Roy 03]

Near-optimal solutions for small k

- $k = 2$, near-linear
  - polygon robot translation [Edelsbrunner-Guibas-Sharir]
  - general [Guibas-Sharir-Sifrony 89]
- $k = 3$, near-quadratic
  - polyhedron robot translation [Aronov-Sharir 88]
  - polygon robot trans+rot [H-Sharir 93]
  - general [H-Sharir 94, Schwarzkopf-Sharir 96]
- $k \geq 4$
  - rod in space, near-optimal [Koltun 05]
  - otherwise, efficient solutions, suboptimal

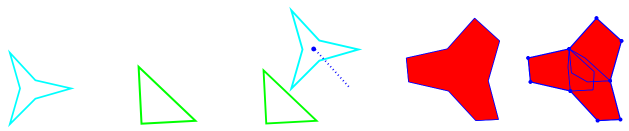
Exact motion planners, based on CGAL arrangements

- polygonal translation w/degeneracies [Flato-H]
- Minkowski sums of convex polytopes [Fogel-H]
  (toward polyhedral translation, effective decomposition [Shaul-H])
- disc among polygons [Leiserowitz], CGAL arrangements with conic traits [Wein]
- optimized translation planning in the plane [Wein-van den Berg-H 05]
- on-going: planar translation and rotation

Minkowski sums and configuration space obstacles

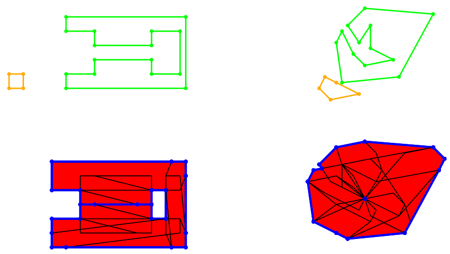
given two planar sets A and B, their Minkowski sum $A \oplus B$ is the set $\{a + b \mid a \in A, b \in B\}$

the robot B collides with the obstacle A whenever the reference point of B is inside the sum $A \oplus (-B)$



Exact Minkowski sums in 2D arrgs of segments

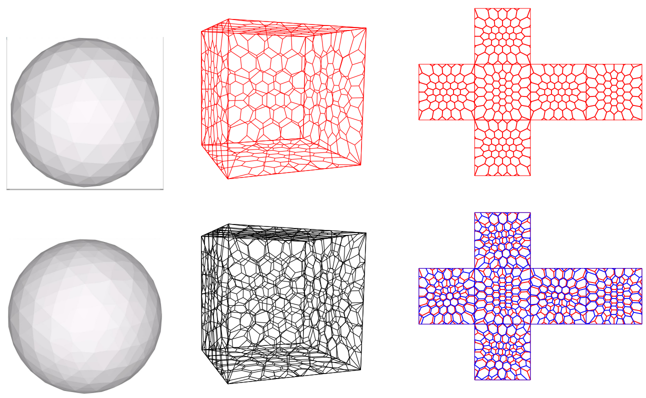
[Flato-H 00]



polygon decomposition has a crucial effect on the efficiency of the Minkowski sum computation [Agarwal-Flato-H 00]

Exact sums of convex polytopes in 3D by overlay of 2D arrgs

[Fogel-H 04]



The primal The CGM The CGM unfolded

Exact motion planners

- polygonal translation w/degeneracies
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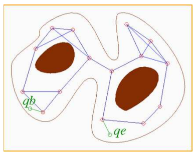
what about more degrees of freedom?

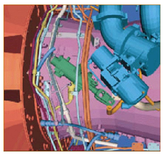
Meanwhile in robotics

- potential field methods [Khatib 80's]
  attractive potential (goal), repulsive potential (obstacles)
- random path planner [Barraquand-Latombe 89]
  combining PF with randomization to escape local minima
- probabilistic roadmaps
  [Kavraki-Svestka-Latombe-Overmars 95]

PRM: probabilistic roadmaps [figures: Latombe's group]



- the approach
- easy to implement, provided you have a good static collision detector
- extended the applicability of motion planning: animation, docking motion, virtual prototyping



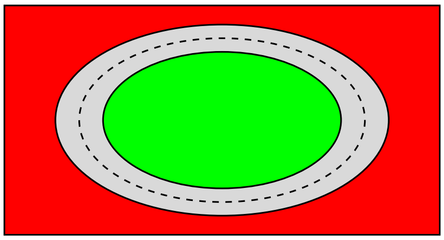
e.g., assembly maintenance

PRM, difficulties

- unpredictability
- narrow passages

Hybrid motion planning

combining exact arrangements with PRM

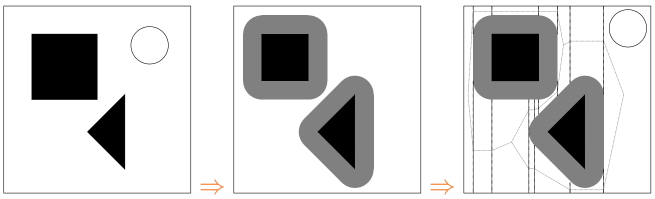


Hybrid motion planning

coordinating two discs moving among polygonal obstacles

[Hirsch-H 02]

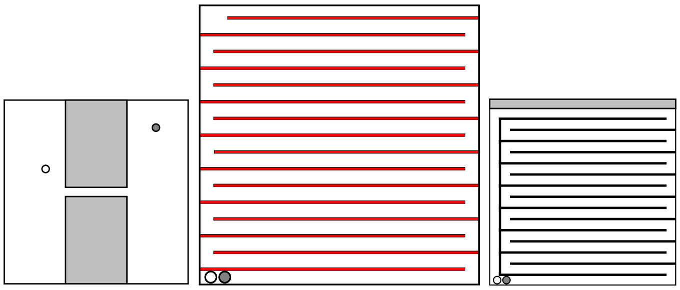
(1) exact arrangement for each disc



(2) Cartesian product of 2D free cells to create 4D cells

(3) PRM inside each 4D cell and between 4D cells

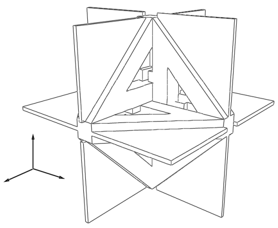
Hybrid motion planning, results



More applications of arrangements

- aspect graphs
- assembly planning, the motion-space approach
- part orienting
- casting
- facility location
- molecular modeling
- . . .

recall Problem 3: Is the 3D object interlocked?

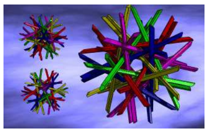


decide whether an assembled object is interlocked, namely cannot be taken apart with two hands

Polynomial time solution

[Guibas-H-Hirukawa-Latombe-Wilson ’98]

- the motion space: space of all possible motions of a subassmebly
- decompose this space by blocking relations between parts
- build the Dirceted Blocking Graph for each cell and check for strong connectivity
- in the case of infinitessimal motion (Problem 3—five-dimensional space): use maximally covered cells



[Snoeyink-Stolfi ’93]

Challenges, summary

- rounding arrangements
- certified computing with finite precision
- effective decomposition of arrangements: theory and practice
- exact motion planning with three and more degrees of freedom
- hybrid techniques
- for a variety of open problems, in both theory and practice, see the 2004-CRC-handbook chapter on arrangements

References

- Robot Motion Planning, Latombe, Kluwer, 1991
- CRC Handbook (Goodman-O'Rourke) chapters on:
  - Algorithmic Motion Planning, Sharir
  - Robotics, Halperin-Kavraki-Latombe
- The surveys cited in the bibliography of the arrangements overview
- www.give.nl/movie/

Solutions to Problems 1–3 of the arrg overview

- **(1)** D. Halperin and M. Sharir. A near-quadratic algorithm for planning the motion of a polygon in a polygonal environment. *Discrete Comput. Geom.*, 16:121–134, 1996.
- **(2)** M. de Berg, D. Halperin, M.H. Overmars, and M. van Kreveld. Sparse arrangements and the number of views of polyhedral scenes. *Int. J. Comput. Geom. Appl.*, 7:179–195, 1997.
- **(3)** L.J. Guibas, D. Halperin, H. Hirukawa, J.-C. Latombe, and R.H. Wilson. Polyhedral assembly partitioning using maximally covered cells in arrangements of convex polytopes *Int. J. Comput. Geom. Appl.*, 8:179–199, 1998.

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