Assignment: Snap Rounding

Do not submit

The topic of this assignment is *snap rounding arrangement of segments*, which we call SR for short, as preparation for the next class. We give a definition of what SR is followed by a several questions on the process and its results. Consider these questions in preparation for the class on SR to be given on April 20th, 2005.

Given a finite collection $\mathcal{S}$ of segments in the plane, the arrangement of $\mathcal{S}$, denoted $\mathcal{A}(\mathcal{S})$, is the subdivision of the plane into vertices, edges, and faces induced by $\mathcal{S}$. A *vertex* of the arrangement is either a segment endpoint or the intersection of two segments. Given an arrangement of segments whose vertices are represented with arbitrary-precision rational coordinates, SR proceeds as follows. We tile the plane with a grid of unit squares, *pixels*, each centered at a point with integer coordinates. A pixel is *hot* if it contains a vertex of the arrangement. Each vertex of the arrangement is replaced by the center of the hot pixel containing it and each edge e is replaced by the polygonal chain through the centers of the hot pixels met by e, in the same order as they are met by e. See Figure 1 for an illustration.

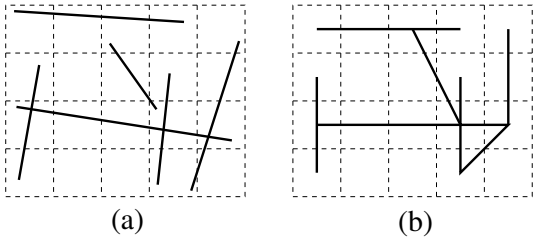

Figure 1: An arrangement of segments before (a) and after (b) snap rounding.

Questions:

1) Given n segments that intersect at k points what is the maximum combinatorial complexity of their arrangement?

2) What is the maximum complexity of a snap-rounded arrangement? use additional parameters as needed (for example N, the number of hot pixels).

3) What is the maximum total complexity of all the n polygonal chains (each approximating an original input segment)?

4) Describe an efficient algorithm to perform SR and bound its running time?

5) Can you make your algorithm output sensitive? if so, sensitive to what output parameters?

6) What is the minimum distance between two vertices in a snap-rounded arrangement? what is the minimum distance between a vertex and a non-incident edge in a snap-rounded arrangement?