Assignment no. 2

due: December 21st, 2005

Solve **five** out of the following seven exercises.

Exercise 2.1 The *pockets* of a simple polygon are the areas outside the polygon, but inside its convex hull. Let P_1 be a simple polygon with n_1 vertices, and assume that a triangulation of P_1 as well as of its pockets is given. Let P_2 be a convex polygon with n_2 vertices. Show that the intersection $P_1 \cap P_2$ can be computed in $O(n_1 + n_2)$ time. (CGAA Ex. 3.12)

Exercise 2.2 The *stabbing number* of a triangulated simple polygon P is the maximum number of diagonals intersected by any line segment interior to P. Give an algorithm that computes a triangulation of a convex polygon that has stabbing number $O(\log n)$.

Exercise 2.3 Prove that the following polyhedron $\mathcal{P}$ cannot be tetrahedralized using only vertices of $\mathcal{P}$, namely its interior cannot be partitioned into tetrahedra whose vertices are selected from the vertices of $\mathcal{P}$ (see the enclosed figure).[1]

> Let a, b, c be the vertices (labeled counterclockwise) of an equilateral triangle in the xy-plane. Let a', b', c' be the vertices of abc when translated up to the plane $z = 1$. Define an intermediate polyhedron $\mathcal{P}'$ as the hull of the two triangles including the diagonal edges ab', bc', and ca', as well as the vertical edges aa', bb', and cc', and the edges of the two triangles abc and $a'b'c'$. Now twist the top triangle $a'b'c'$ by 30° in the plane $z = 1$, rotating and stretching the attached edges accordingly: this is the polyhedron $\mathcal{P}$.

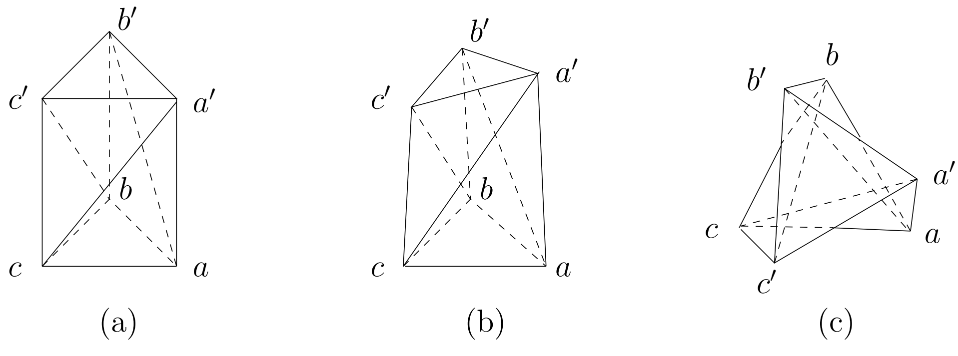

Figure 1: The untetrahedralizable polyhedron is constructed by twisting the top of a triangular prism (a) by 30° degrees, producing (b), shown in top view in (c)

Notice that there are additional exercises on the other side of the page.

[1]This construction is due to Schönhardt, 1928. The description here is taken from O'Rourke's *Art Gallery Theorems and Algorithms*.

Exercise 2.4 Instead of removing the object from the mold by a single translation (as we will show in class), we can also try to remove it by a single rotation. For simplicity let's consider the planar variant of this casting problem, and let's only look at clockwise rotations.
(a) Give an example of a simple polygon P with top facet f that is not castable when we require that P should be removed from the mold by a single translation, but that is castable using rotation around a point.
(b) Show that the problem of finding a center of rotation that allows us to remove P with a single rotation from its mold can be reduced to the problem of finding a point in the common intersection of a set of half-planes.
(CGAA Ex. 4.7)

Exercise 2.5 A simple polygon P is called *star-shaped* if it contains a point q such that for any point p in P the line segment $\overline{pq}$ is contained in P. Give a linear time algorithm to decide whether a simple polygon is star-shaped.

Exercise 2.6 On n parallel railway tracks n trains are going with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.

Exercise 2.7 Describe in detail the procedure UnboundedLP3, which accepts n halfspaces in three-dimensional space, and an objective function. The procedure either finds that the induced LP is infeasible, or that the LP is bounded in which case UnboundedLP3 outputs three witnesses to this fact, or outputs a ray (in three-dimensional space) such that as we proceed away from the ray's terminus, the objective function grows. Show that the procedure runs in time $O(n)$.