Algorithms for 3D Printing and Other Manufacturing Processes

Movable separability II: The motion space approach to assembly planning

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Spring 2017

Outline

- Basics of assembly planning, reminder
- The motion space approach
- Multi-step motions
- Infinitesimal separability
- Challenges

Basics, reminder

Assembly by disassembly

- Partitioning, first the input assembly *A* into subassemblies and then, recursively, the generated subassemblies that are not individual parts
- Implemented by two procedures:
- Partition: takes the description of an assembly *S* as input and generates two subassemblies S_1 and S_2, along with a path p such that moving S_1 along p separates it from S_2. Whenever such subassemblies and direction do not exist, the procedure returns failure
- Disassemble: applies partition to the given assembly *A* and, recursively, to the generated subassemblies

Categories of motion

- A motion step translates a body along a single direction t by some distance q, while rotating it at constant rate r about an axis a that is fixed relative to the body
- A **one-step motion** consists of a single motion step in which q is arbitrarily large
- If the rotation rate is null (i.e., the body has fixed orientation), the motion is a **one-step translation**
- A **multistep motion** is the concatenation of several motion steps, in which the last step has arbitrarily large q
- An **infinitesimal motion** consists of a single motion step in which q is arbitrarily small

The motion space approach

A general framework

- A general approach to designing the procedure partition
- The procedure needs to select a subset S out of A (exponentially many options) and a path p along which we separate S from A\S
- The crux of the framework is the observation that the number of degrees of freedom of the path is the key factor of efficiency
- A suite of polynomial-time solutions to the partitioning problem and hence to assembly planning

Motion space (M-space)

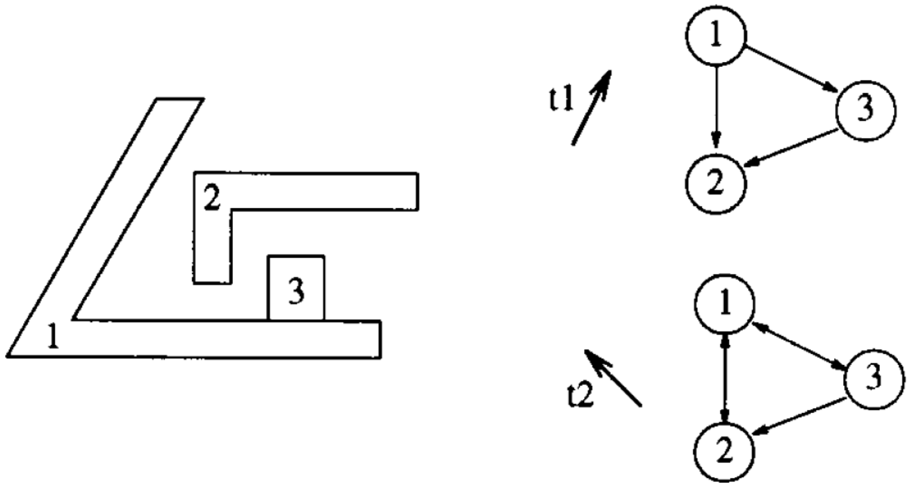
- the space of parametric representations of all allowable motions for partitioning operations: every point in M-space uniquely defines a path of the subassembly moved by an operation
- the dimension of the motion space is the minimal number of parameters required to define a path with a fixed starting point
- the motion space must be parameterized in such away that the representation of a motion is independent of the subassembly that will eventually be moved away
- all the coordinate frames coincide with a universal frame U when the parts are in their assembled configurations

M-region

- For every ordered pair of parts P_i and P_j in an assembly we define their M-region P_{ij} to be the collection of points p in motion space such that if we move P_i along the path that p represents, P_i will overlap with P_j at some point
- For each path p in P_{ij} we say that P_j **blocks** P_i

Blocking graph

- Given an assembly A made of n parts $P_1,\ldots,P_n$, we associate a directed graph G(p) with every point p of motion space. The nodes of G(p) are the n parts $P_1,\ldots,P_n$ composing the assembly, and each ordered pair $<P_i,P_j>$ such that $p \in P_{ij}$ induces an arc of G(p) directed from P_i to P_j .We call this graph the directional blocking graph (or DBG) of A for path p



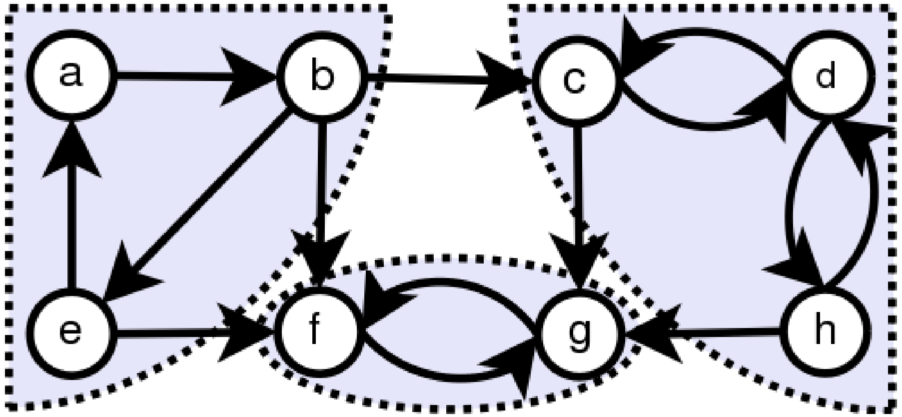
Non-directional blocking graph

[Wilson-Latombe ‘94]

- Let ∂P_{ij} denote the set of all paths *p* such that if P_i moves along *p* it will eventually touch P_j, without overlap
- In general ∂P_{ij} is a superset of the boundary of P_{ij}
- The sets ∂P_{ij} for all $i,j \in [1,n]$, $i \neq j$, decompose the motion space into an arrangement of cells such that the DBG of *A* remains fixed over each cell
- The arcs of the DBG in any cell c in this arrangement correspond exactly to the M-regions that contain c
- The arrangement of cells thus defined, along with the DBG of each cell, the **non-directional blocking graph** (or NDBG)

Strong connectivity

- A directed graph is strongly connected if every vertex is reachable from every other vertex
- The strongly connected components of an arbitrary directed graph form a partition into subgraphs that are themselves strongly connected
- It is possible to test the strong connectivity of a graph, or to find its strongly connected components, in linear time



Graph with strongly connected components marked [Wikipedia]

NDBG and the partition problem

- Claim: The assembly partitioning problem has a positive answer (subassembly + path) iff there is a point p in M-space such that DBG(p) is not strongly connected

The procedure partition

procedure `partition`(S);
    for every cell c in the NDBG of S **do**:
        if the DBG associated with c is not strongly connected
            then return c and a feasible partition of S;
    return failure;

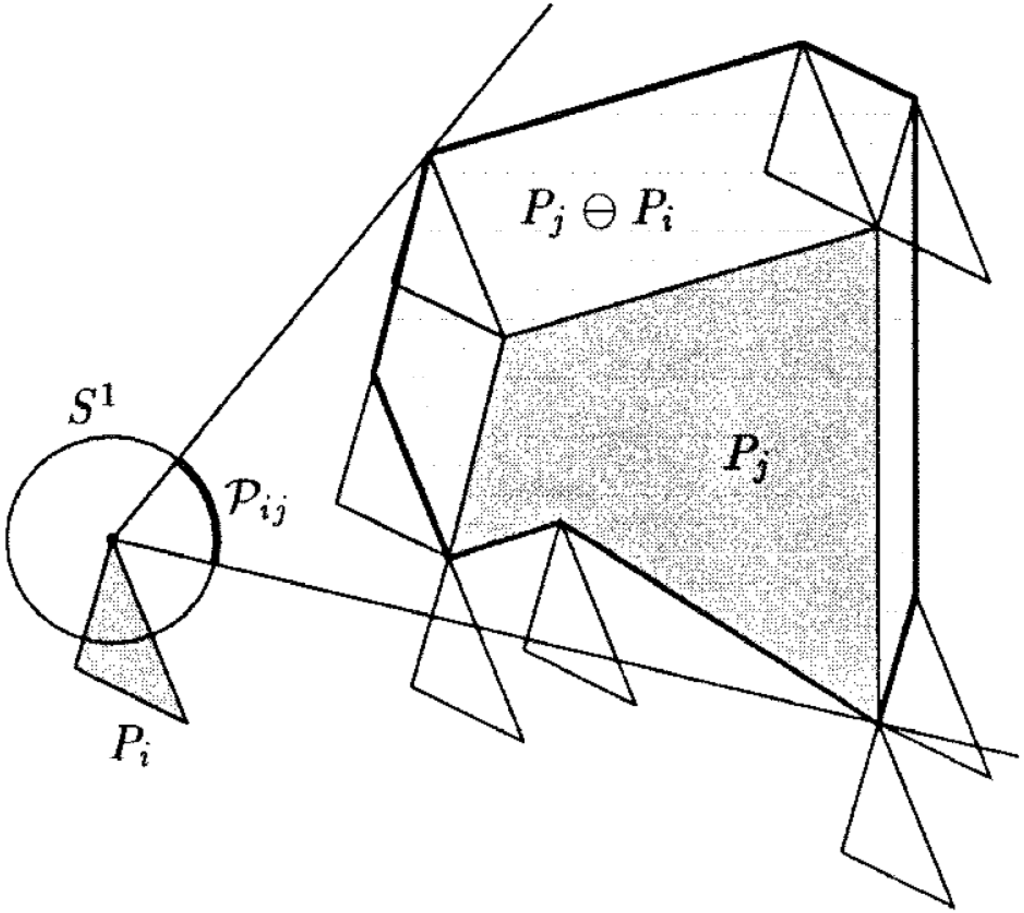
Motion space realization

- Define the parameters of a partition path
- This determines the dimension d and coordinates of the M-space
- The shape and complexity of an M-region
- The overlay of the boundaries of the M-regions: a d-dimensional arrangement
- The rest is (more or less) common to all realizations:
    - Construct the DBG in each cell according to containment in M-regions
    - Check each DBG for strong connectivity

Example 1: One-step translation in the plane

- Reminder I:
  - A **one-step motion** consists of a single motion step in which q is arbitrarily large
  - If the rotation rate is null (i.e., the body has fixed orientation), the motion is a **one-step translation**
- What is the M-space?
- How does an M-region look like?
- Reminder II:
  - Two sets A and B intersect if and only if the Minkowski sum $A \oplus -B$ contains the origin, where $-B$ is the set B reflected through the origin
  - More generally: $A \cap (B \oplus \{t\}) \neq \emptyset$ iff $t \in A \oplus -B$

One-step translation, M-region



One-step translation, constructing the NDBG

- Overlay the arcs P_{ij} on S^1: sort the endpoints of the arcs
- Compute the BDG at, say, θ=0
- * Check for strong connectivity, if not SC, report a strongly connected component and a direction of separation and stop, else
- Move to the next cell of the 1D arrangement. If contains θ=0, then report failure and stop. Else update the DBG according to the vertex you crossed, and go to *

One-step translation, complexity

- n - # of polygonal parts
- q – maximum complexity of a single part
- The boundary of P_{ij} can be computed in $O(q^2)$ time for a total of $O((nq)^2)$ time – needs some care
- Overlay of the n^2 arcs on S^1 results in an arrg of complexity $O(n^2)$ and takes $O(n^2 \log n)$ time to construct
- The complexity of a single DBG is $O(n^2)$ and it takes $O(n^2)$ time to build it
- Updating the graph at a crossing point takes constant time
- Total construction time of the NDBG $O(n^2(\log n + q^2))$

One-step translation, complexity, cont'd

- Deciding strong connectivity for a single DBG takes $O(n^2)$ time
- Total running time of the disassemble procedure $O(n^5)$
- Total running time of the entire algorithm $O((nq)^2 + n^5)$

Amortizing strong connectivity tests

- One can use the knowledge about the sequence of insertions and deletions of edges in all the DBGs together to improve the amortized running time of a strong-connectivity test to $O(n^{1.376})$ [Khanna-Motwani-Wilson ‘98]

One-step translation in 3-space

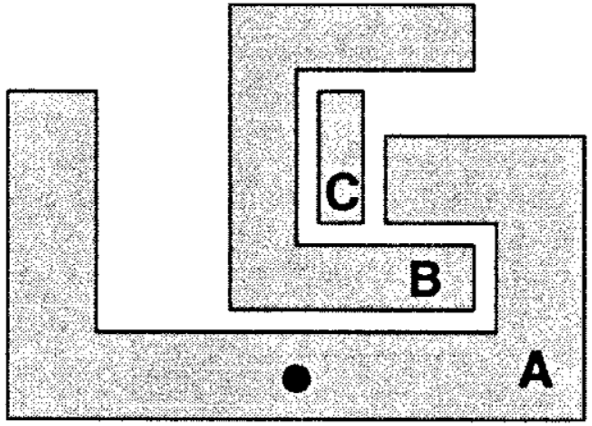
- See separate set of slides

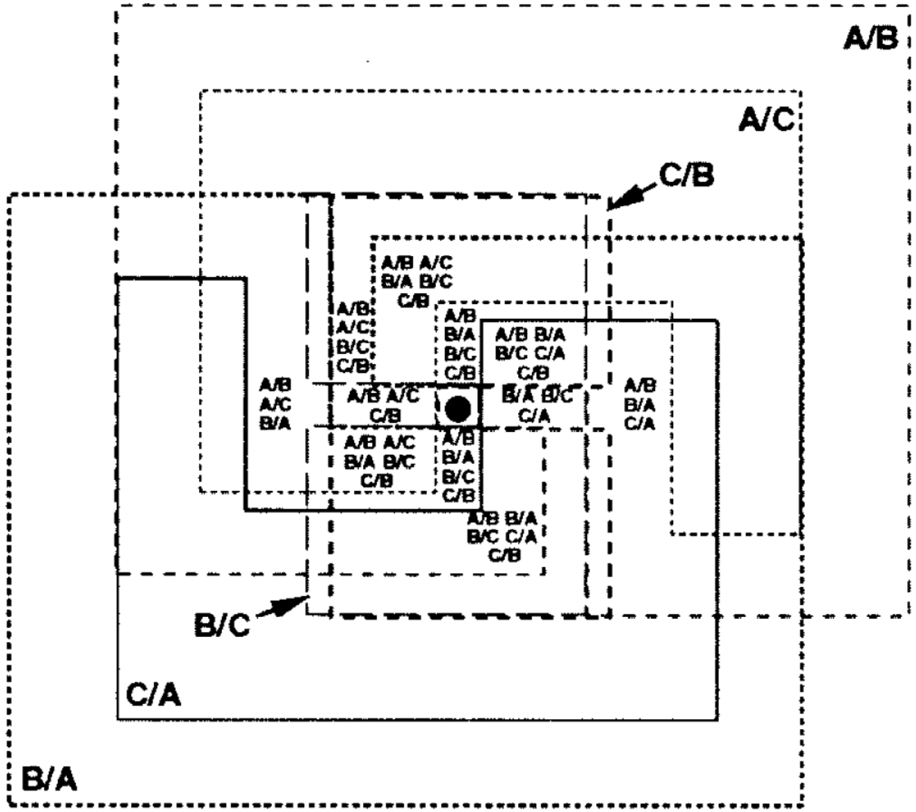
Multi-step motions

Finite set of arbitrary-length multi-step paths

- If we are given the subassembly, finding a multi-step path can be carried out in polynomial time
- If the assembly is not given but the given infinite family of paths (M-space) has fixed dimension, then the partition problem can be solved in polynomial time
- Question: We are given an assembly and we have to solve the partition problem for a finite set of arbitrary-length multi-step paths; can we solve this problem efficiently, say even for multi-step translations in the plane?

The interference diagram (ID) for translations

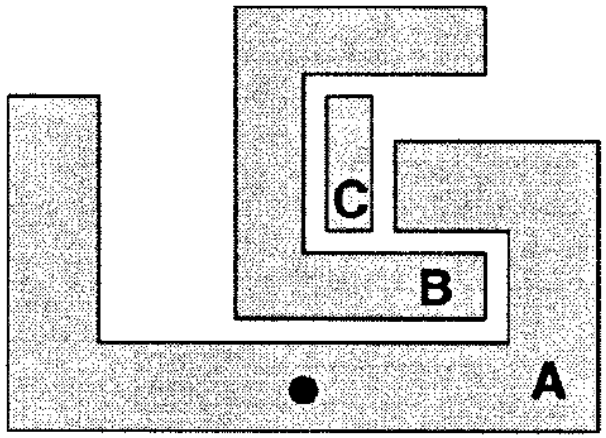


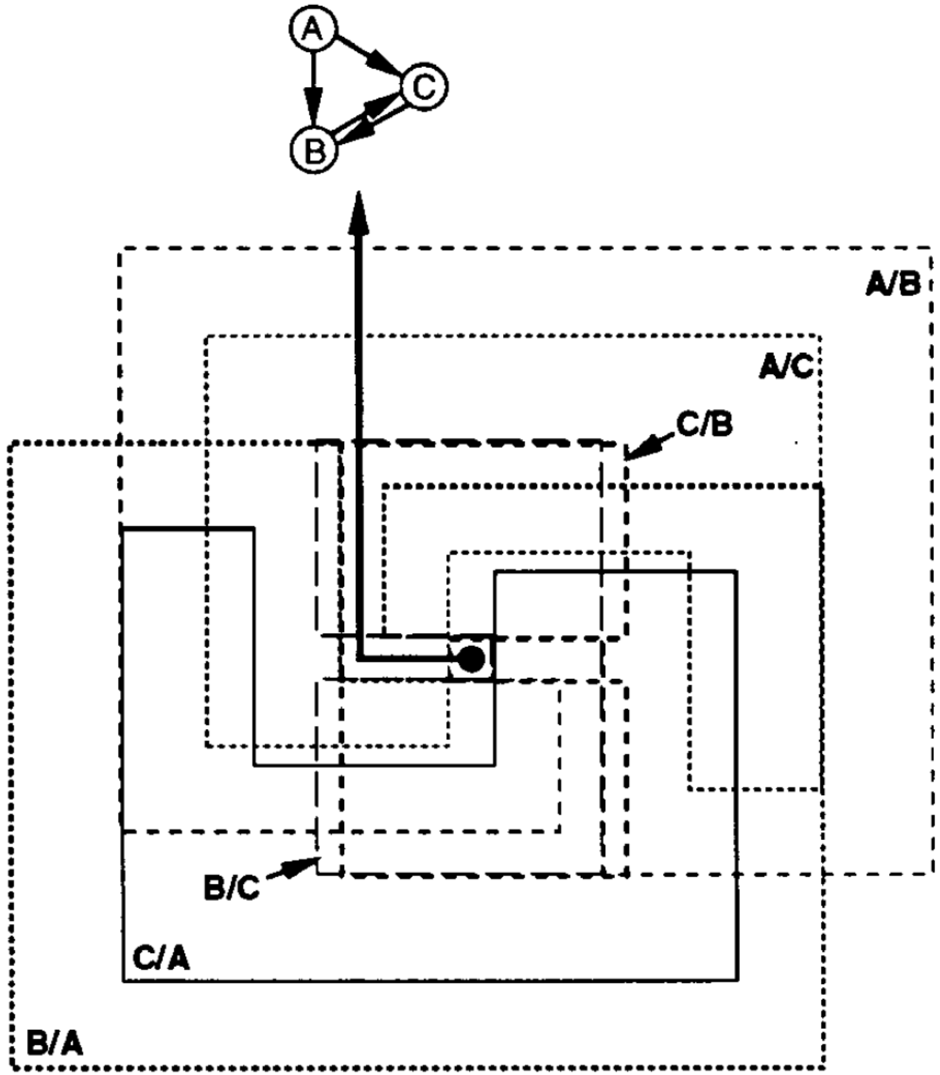


ID, details of multi-step translation in the plane

- The assembly is placed in a fixed location in the plane
- The assembly and all its parts have a joint reference frame
- We compute the Minkowski sum $P_j \oplus -P_i$ for each ordered pair of distinct parts
- Next, we construct the arrangement of the boundaries of the Minkowski sums
- Each valid path starts at the common origin and ends at the unbounded cell of the arrangement
- The DBG of the path has an edge (P_i,P_j) for every $P_j \oplus -P_i$ that it crosses

Example





Suggestion for a project

- Devise an interactive graphic program to answer the partition problem for query multi-step-translations paths for polygonal parts in the plane. Analyze the complexity of each step.

Remarks:

- The ID is given almost for free with CGAL (Minkowski sums + arrangements of segments)
- Challenge 1: construct an efficient version of the ID (not all details in a Minkowski sum may be necessary)
- Challenge 2: allow for tight passages in the partition paths

ID for multi-step motions (trans+rot) in the plane

- What is the shape of an M-region?
- How can we construct it?

- C-space visualization:

https://www.youtube.com/watch?v=SBFwgR4K1Gk&feature=youtu.be&hd=1

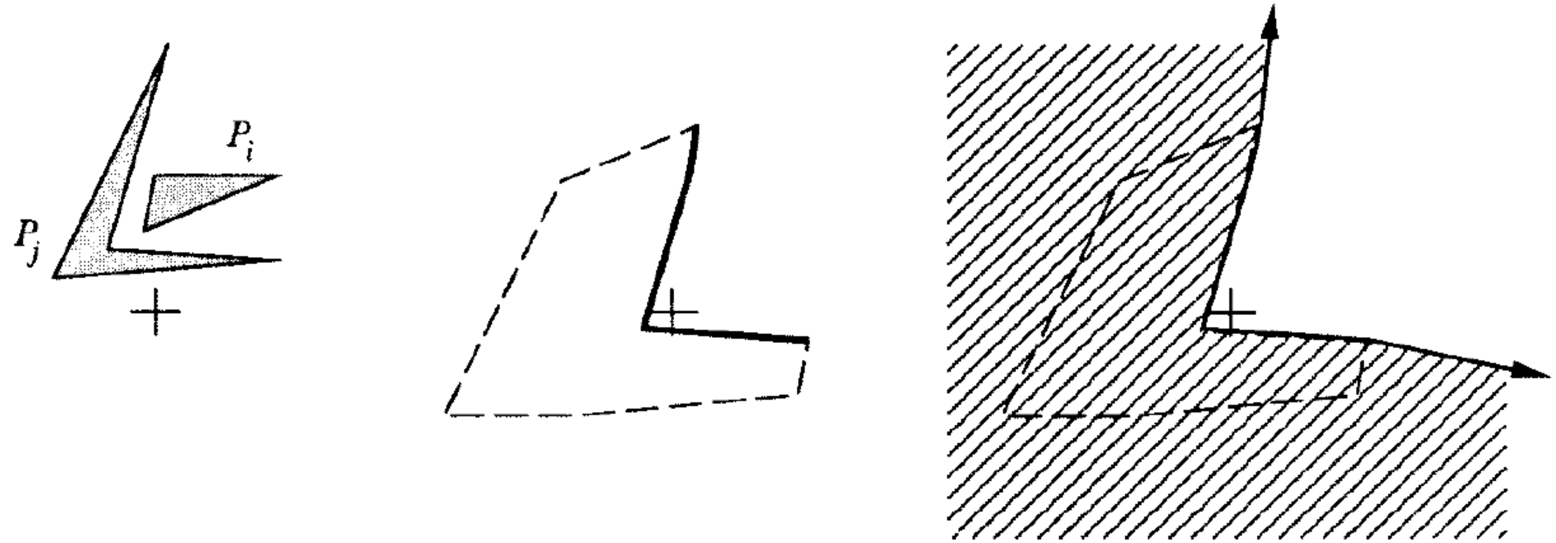
Two-step translations in the plane: The M-space approach

- Every partition path consists of a segment from the origin to the point (x,y) followed by a ray in direction θ
- The M-space is three-dimensional, with coordinates (x,y,θ)
- We will construct a superset of the boundary surfaces of the M-regions
- Once we have the arrangement of these surfaces, the procedure is as before

The boundary surfaces of the first step

- Consider first only paths that start at the origin and end at the point (x,y); the M-region P_{ij}:

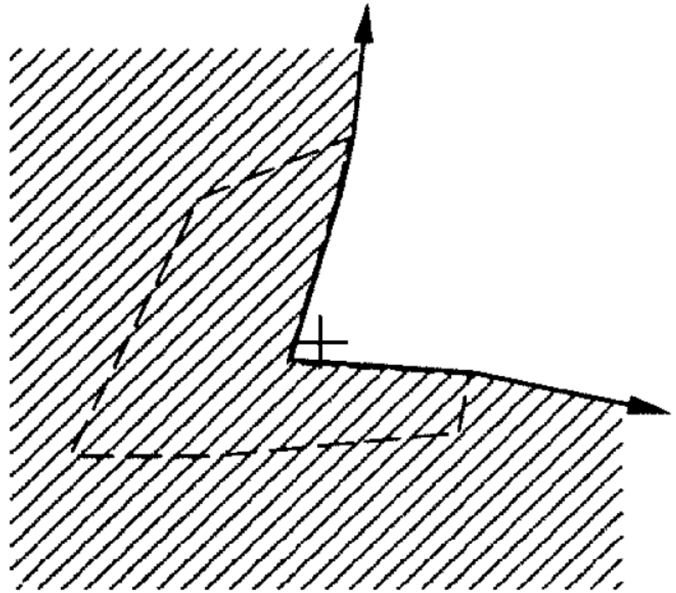
(x,y) is part of the M-region if P_i, when moved along the segment from the origin to (x,y), intersects P_j



- Notice the difference between the contribution of P_i,P_j to the ID (middle figure) and to M-space (right figure)

First segment contribution to the entire M-space

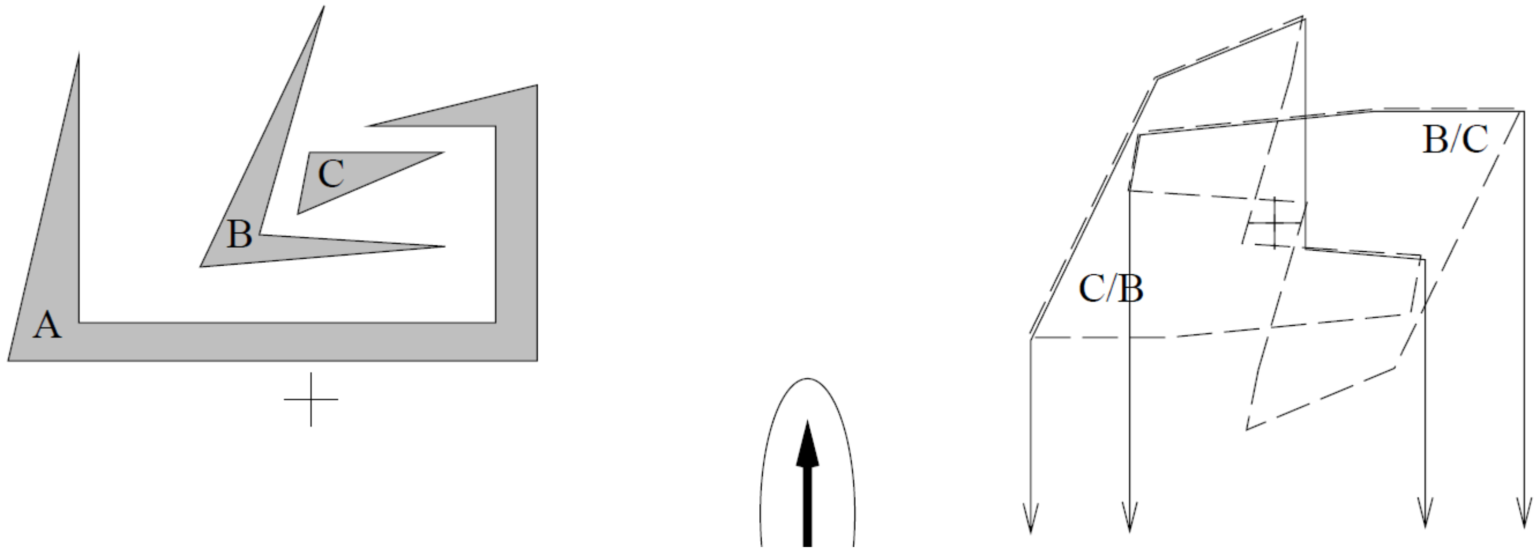
- Notice that the portion of the M-region of P_{ij} due to the first segment remains the same regardless of what the direction of the final ray is
- We extend the boundary curves in the plane to be the same for every θ-slice



The boundary surfaces of the second step

- Consider now only paths that start at (x,y) and move to infinity along the direction θ; an θ–slice of the M-region P_{ij}:

(x,y) is part of the θ–slice of the M-region P_{ij} if P_i, when moved along the segment from (x,y) along θ to inifinity, intersects P_j



Second, ray, contribution to the entire M-space

- We now have to make similar analysis for every direction θ between 0 and 2π, and take the union of all these curves
- Distinguish between shadows that are part of the Minkowski sum and shadows that are rays and produce the respective surfaces separately

[H-Wilson ‘96]

Infinitesimal separability in 3-space

Leonidas J. Guibas, Dan Halperin, Hirohisa Hirukawa, Jean-Claude Latombe, Randall H. Wilson: Polyhedral Assembly Partitioning Using Maximally Covered Cells in Arrangements of Convex Polytopes. Int. J. Comput. Geometry Appl. 8(2): 179-200 (1998)

Preliminaries

- The direction of a one-step motion is given by a unit vector in six dimensions
- An infinitesimal motion separates two subassemblies if it displaces one relative to the other, by an arbitrarily small amount, without overlapping of their interiors (modified from our earlier goal of “sufficiently far away from one another”)
- Makes sense only if the assembly is connected

Motivation

- Much easier than full one-step motion in 3-space - M-space is 6-dimensional with complex shapes of M-regions
- Infinitesimal motion can be used as a hint for finite motion – continued in the same direction
- Testing if an object is interlocked – no infinitesimal motion, then there is no separation with two hands

M-space

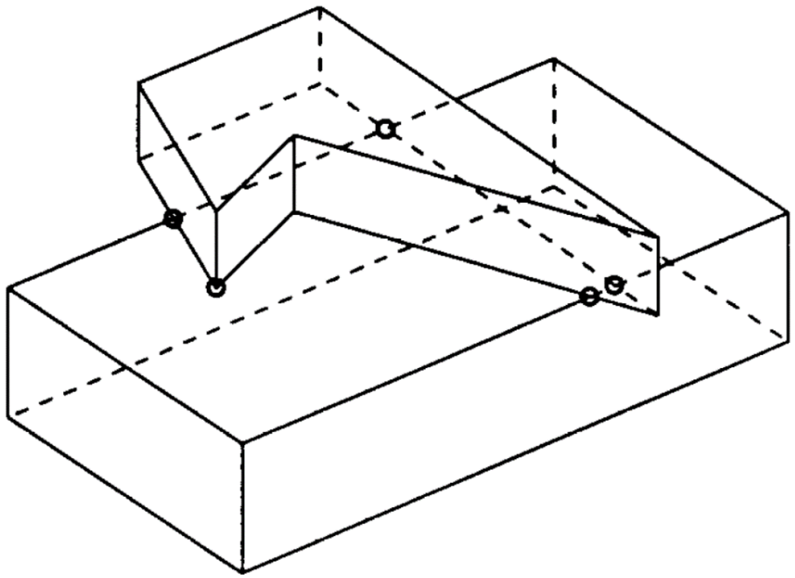
- As before, all parts are represented in a common universal coordinate frame U
- We represent all possible infinitesimal motions on S^5, the (five-dimensional) unit sphere in six-dimensional space

The kinematics of contacts (“black box”)

- An infinitesimal motion of any part Pi is described by a six-dimensional vector dX=(dx, dy, dz, da, db, dc): three components for translation and three components for rotation
- Let v be a vertex of P_i. The motion described by dX causes v to undergo a translation $J_v dX$, where J_v is a constant 3x6 Jacobian matrix: each column of J_v gives the translation v experiences due to a unit motion of P_i in the corresponding parameter of dX
- Assume that P_i and P_j are in contact such that the vertex v of P_i is contained in the face f of P_j. Let n_f be the outgoing normal vector to f. The motion dX causes v to penetrate f when $n_f J_v dX < 0$, to break the contact with f (> 0), slide in f (=0)

The kinematics of contacts, cont'd

- Let P_i and P_j be two parts in contact such that a face f of one and a face g of the pother are overlapping
- Let ui be the vertices of the convex hull of f∩g
- Claim: The set of allowable motions for this contact are the intersection of the closed half-spaces $n_g J_{ui} dX \geq 0$



The critical surfaces of the M-space

- For each vertex ui of the convex hull of the intersection of two parts, the equation $n_g J_{ui} dX = 0$ defines a five-dimensional hyperplane in the six-dimensional space of infinitesimal motions, which partitions S_5 into two open half-spheres and a great circle
- These induce an arrangement of cells of dimensions 0,1,2,3,4,5 on S^5. The DBG is fixed over each such cell
- After constructing the NDBG (5D arrg + DBGs), the rest is as before

Infinitesimal motion in 3-space, complexity

- n - # of polygonal parts
- Two faces with complexity f and g can have O(fg) intersections but only O(f+g) vertices on the boundary of the nvex hull of the intersection
- K – # of ordered pairs of parts in contact
- N- # of contact-constraints (penetration)
- The size of the NDBG arrangement is $O(N^5)$
- Each DBG has $r \in O(n^2)$ edges, where r is the number of pairs of parts in contact
- Computing a candidate partition takes $O(rN^5)$
- We will improve on this procedure below

Complexity

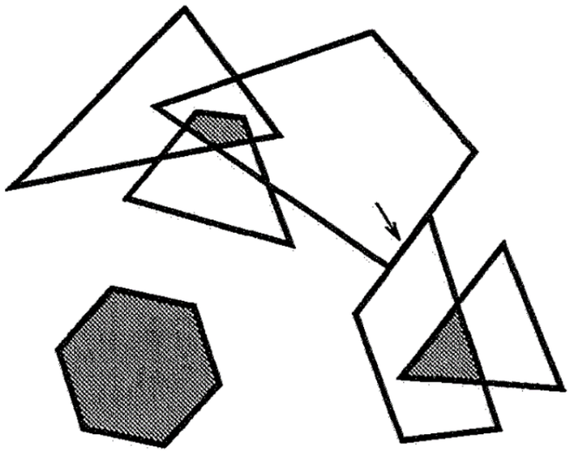
- Given n parts in the assembly, let D1 and D2 be two DBGs for a certain motion space, Di=(V,Ei), with E1⊆E2, the it suffices to test D1 for strong connectivity (monotonicity of strong connectivity)
- Ideally, we would know this containment property without even computing E2. Then this would save not only the test for SC but also the construction of D2
- We manage to exploit this observation for infinitesimal motions. We exemplify thos for infinitesimal translation in 2-space

Infinitesimal translations in 3-space

- The M-space is S^2
- We project it on a plane tangent to the sphere: the information on the two hemispheres is symmetric
- Let Q_{ij} denote the complement of P_{ij}: every point in Q_{ij} represents an infinitesimal transaltion where P_i does not penetrate in P_j

Maximally covered cell

- Maximally covered cell: a cell that is covered by more Q_{ij}'s than its immediate neighbors



- Claim: it suffices to test only the DBGs of maximally covered cells for strong connectivity

Maximally covered cells: number and algorithm

- Given an assembly A of n polyhedral parts, the NDBG of the assembly for infinitesimal motions, where any direction of motion is defined by *d* parameters, has at most $O(K^d)$ maximally covered cells, where K is the number of ordered pairs of parts in contact in the assembly. A sample direction in each maximally covered cell can be computed in total time $O(K^{d-1}N)$ time, where N denotes the number of "equivalent" point–plane contacts in the assembly, as before
- Compare with the earlier result of $O(N^5)$ – in practice $K \ll N$ – see table in the next slides
- The cells are sampled using linear programming – much easier than computing arrangements in higher dimensions

[Guibas et al '98]

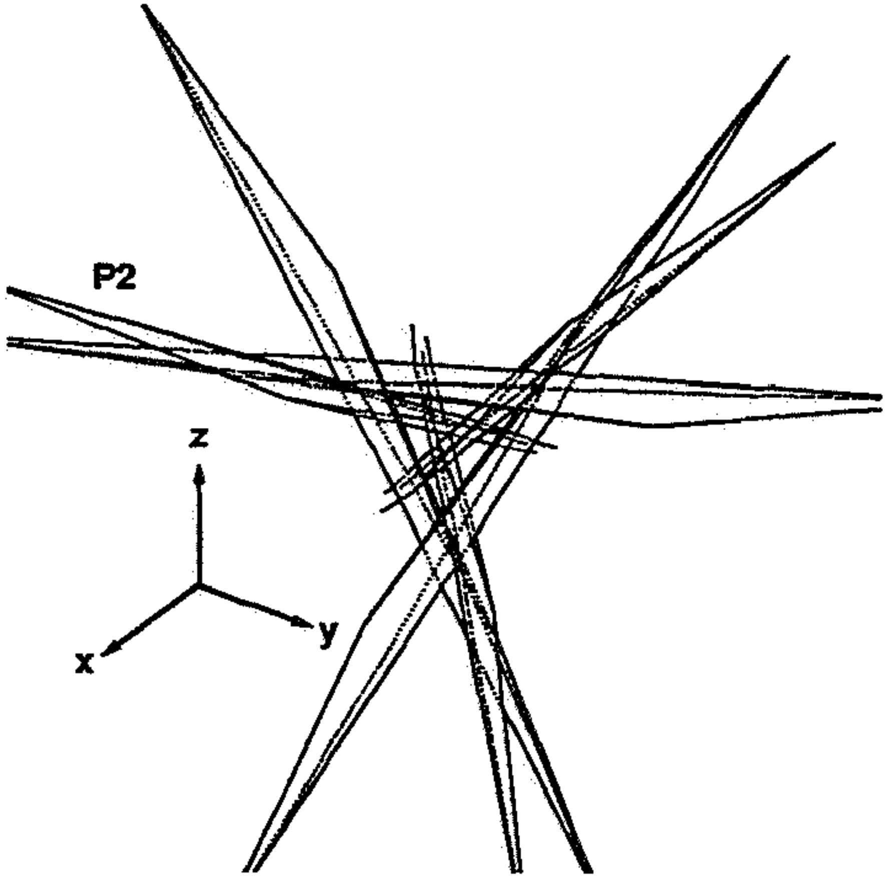


Fig. 11. Six tetrahedra in contact[19]

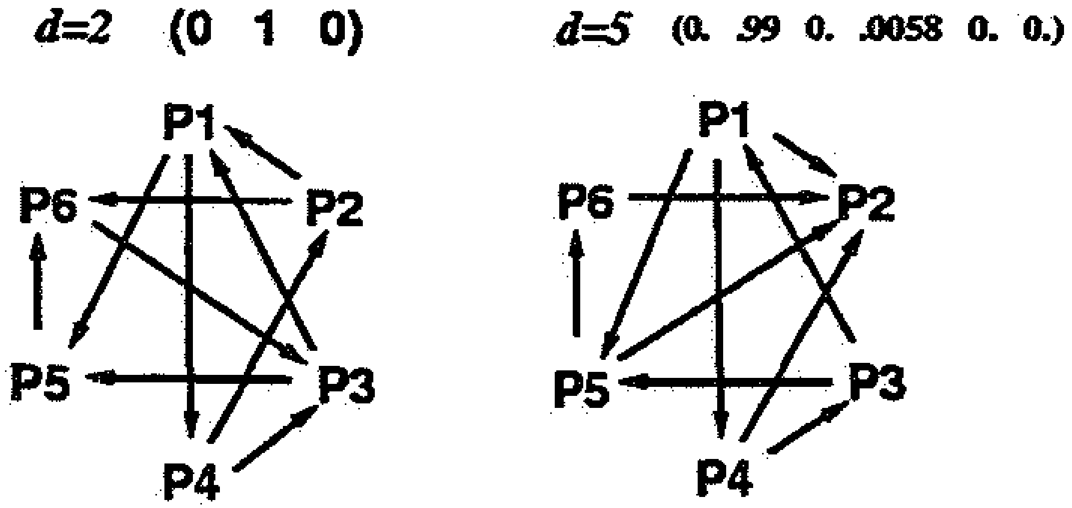


Fig. 12. Examples of the DBGs for the six tetrahedra

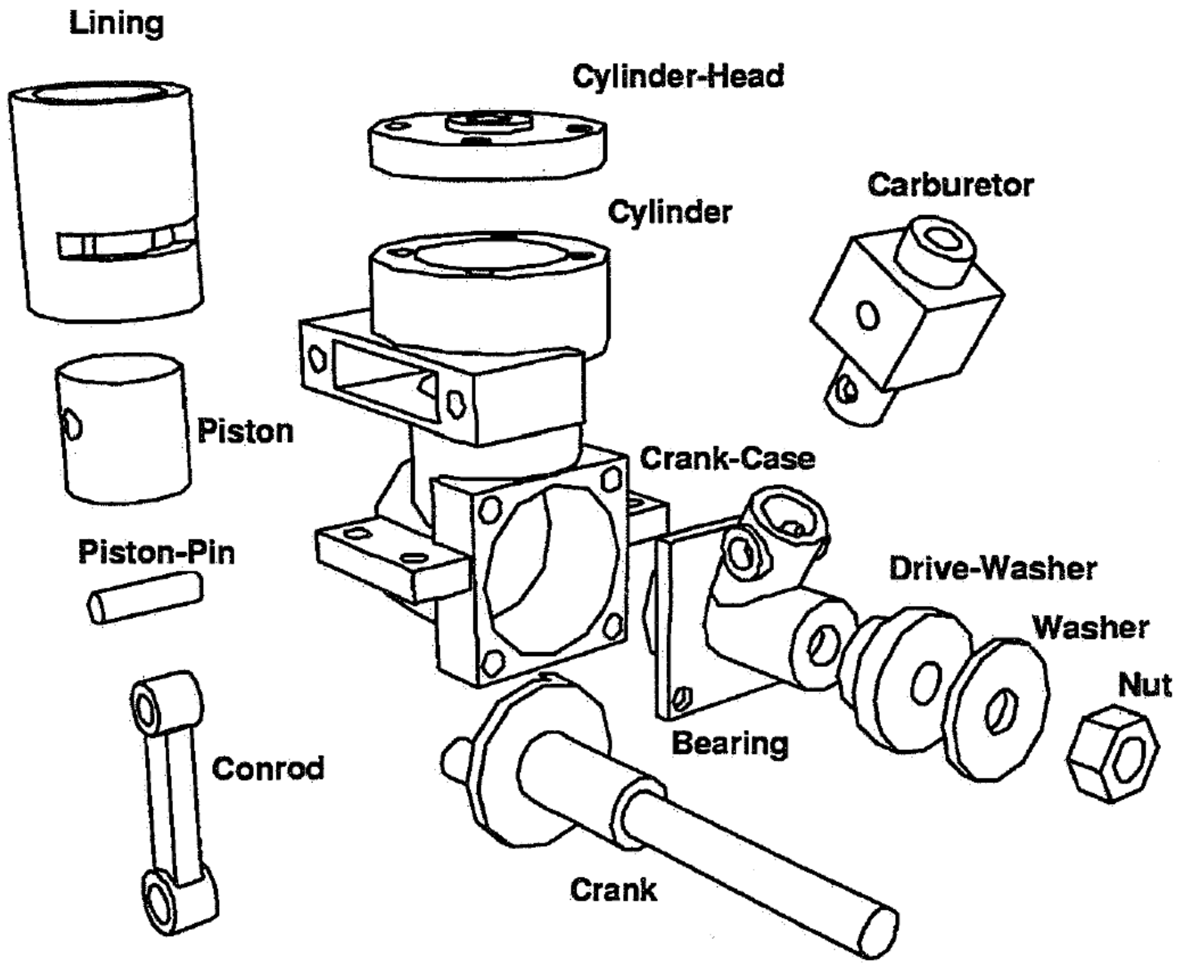


Fig. 13. Parts of a model-aircraft engine

of the parts is 12, the total number of their faces is $1,066$, $K = 24$, and $N = 1,112$. Note that the cylindrical surfaces are approximated by polygonal ones, and that K is not much larger than the number of the parts.

Table 1. Summary of experiments, CPU time reported in seconds

	Puzzle	Assembly	SS-6*	SS-6	SS-30	Engine	Engine
d	2	5	2	5	2	2	5
K	24	6	24	24	300	24	24
N	352	140	96	96	1,200	1,112	1,112
# of Rep. Pts.	3	3	22	190	2,838	4	6
# of S.C. DBGs	2	2	22	118	2,838	0	0
# of Parts	6	3	6	6	30	12	12
Time for Constraints	104.8	12.3	4.0	4.0	808.0	1881.7	1881.7
Time for Rep. Pts.	4.7	2.8	2.7	1286.9	509.0	12.6	7880.1
Time for DBGs	12.7	4.7	27.0	240.5	53,021.3	59.9	120.3

* SS-6 refers to the Snoeyink-Stolfi example with six parts.

Challenges

- k-handed assembly planning, for $k>2$
- Constraints introduced by the mechanical system manipulating the parts (moving and holding the parts together)
- Assembly planning for tolerance parts
- Optimizing assembly sequences

Separability: Bibliography

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Separability: Bibliography, cont'd

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THE END

[Gaither, ArtByAI, CGAL arrgs]