APPLIED aspects of COMPUTATIONAL GEOMETRY

Minkowski Sums, Preliminaries

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Overview

- what are Minkowski sums
- basic properties
- the case of two convex polygons

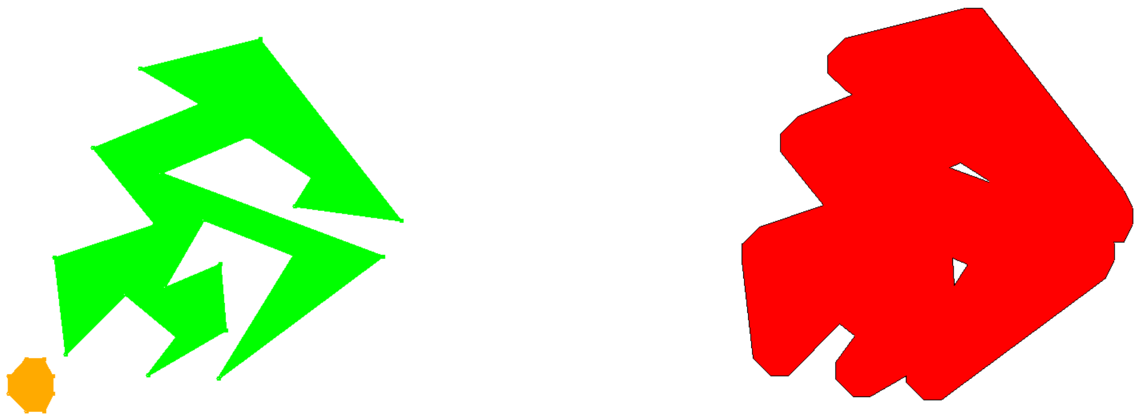
later:

- convex polyhedra
- arbitrary polygons
- offset polygons

- arbitrary polyhedra

Minkowski sums

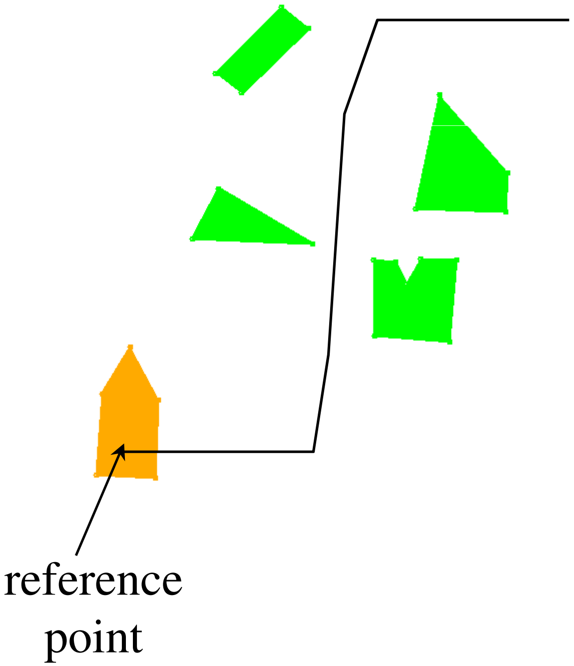
- Given two sets P and Q in R^d, their *Minkowski sum* $P \oplus Q = \{p+q \mid p \in P, q \in Q\}$

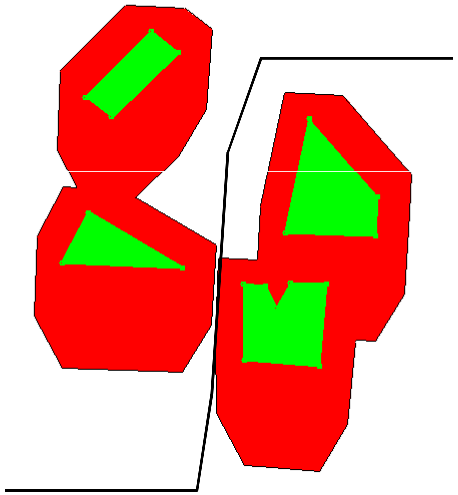


Typical usage: collision detection

R - a polygonal object that moves by translation

P - a set of polygonal obstacles





Claim: When translating, R intersects P iff ref(R) is inside $P \oplus -R$

Typical usage, cont'd

R - a robot that moves by translation

P - a set of obstacles

Claim: When translating, R intersects P iff ref(R) is inside $P \oplus -R$

R(x,y) denotes R with the reference point at (x,y)

$-R = -R(0,0) = \{(-a,-b) \mid (a,b) \in R\}$

Proof

(not restricted to 2D)

Basic properties

- the Minkowski sum of two convex sets is convex
- a farthest point in direction d in the Minkowski sum $P \oplus Q$ is the sum of a farthest point in direction d in P and a farthest point in direction d in Q

➡ the Minkowski sum of convex polygons is a convex polygon

Sum of convex polygons

- $|P|=m$ $|Q|=n$
- The complexity (# of edges here) of the sum is at most

  $m+n$
- when is it exactly $m+n$?
- Algorithm
  - the normal diagram of a convex polygon
  - 1D arrangement overlay
  - running time $O(m+n)$

Minkowski sum of arbitrary polygonal sets

- notice that $P_1 \oplus (P_2 \cup P_3) = (P_1 \oplus P_2) \cup (P_1 \oplus P_3)$

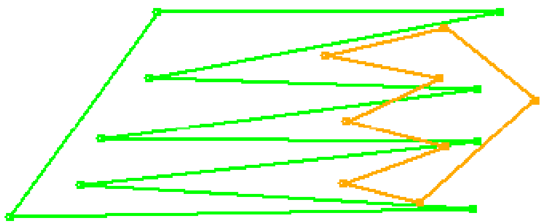
- Step 1 Decompose P and Q into convex subpolygons $P_1, \ldots, P_s$ and $Q_1, \ldots, Q_t$
- Step 2 Compute $P_i \oplus Q_j$ for each pair
- Step 3 Construct the union of those subsums

➡ the Minkowski sum of polygonal sets is a polygonal set

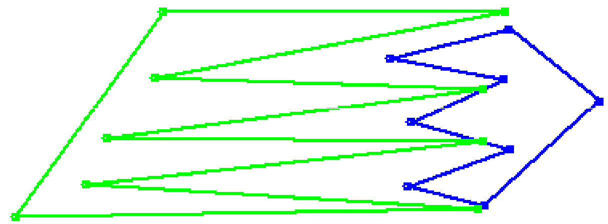
(more implications, on complexity and algorithms, will follow)

Applications of Minkowski sums, reminder: Minimum distance separation

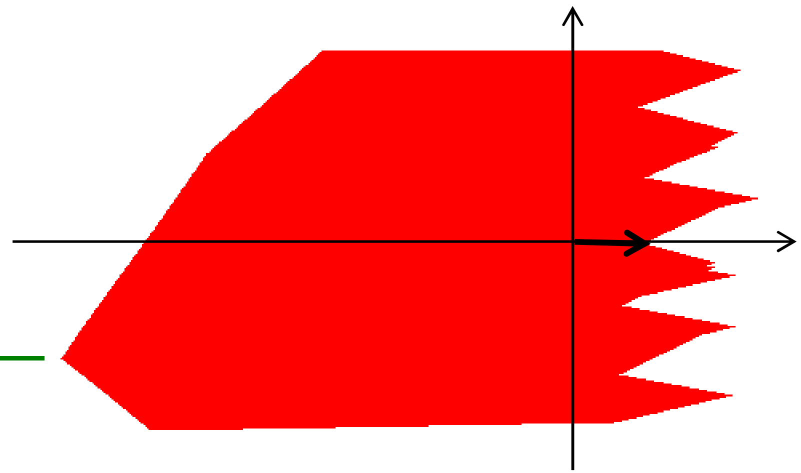
Translate the small polygon P such that it will not penetrate Q



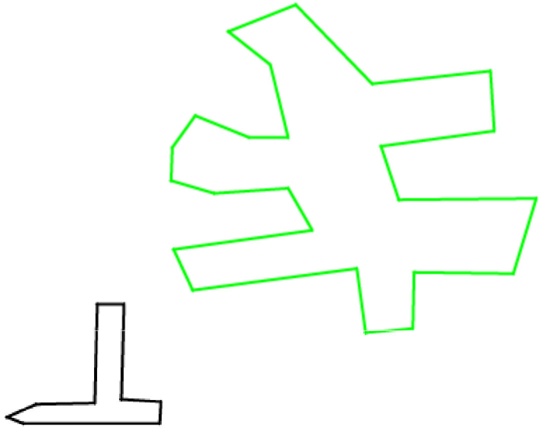
Separated polygons

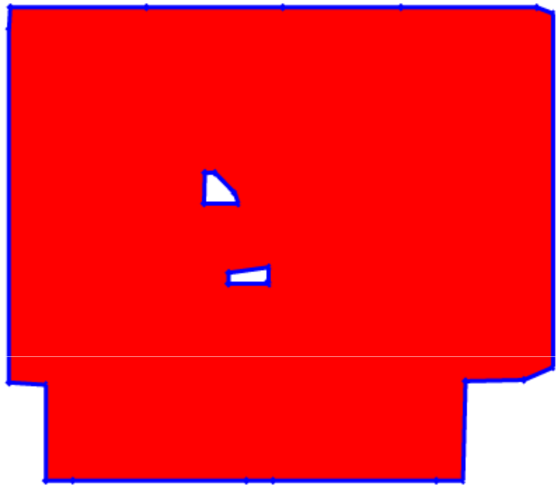


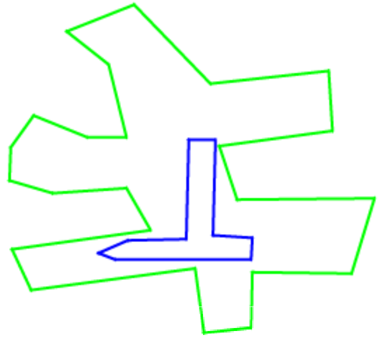
Find the closest point to the origin that is outside $Q \oplus -P$



Applications of Mink sums, cont'd: Polygon containment



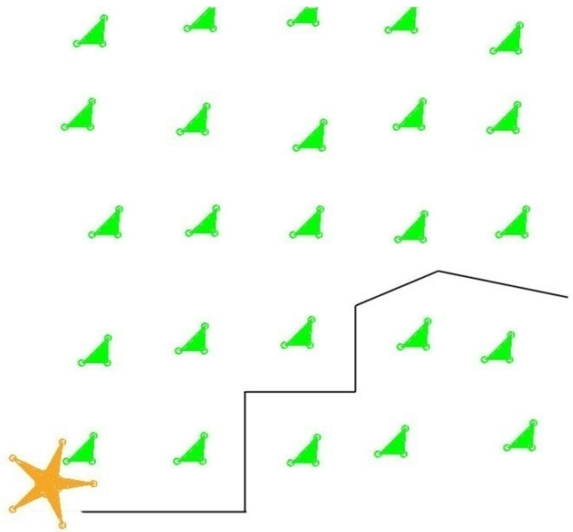




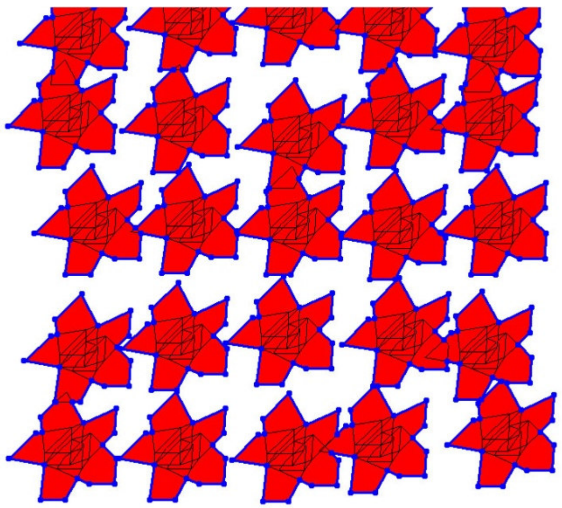
Can a polygon *P* be placed inside another polygon *Q*?

Compute **(*B* \ *Q*) ⊕ -*P***:
(*B* is a bounding box of *Q*)
P can be placed inside *Q* when the reference point is placed in one of the holes

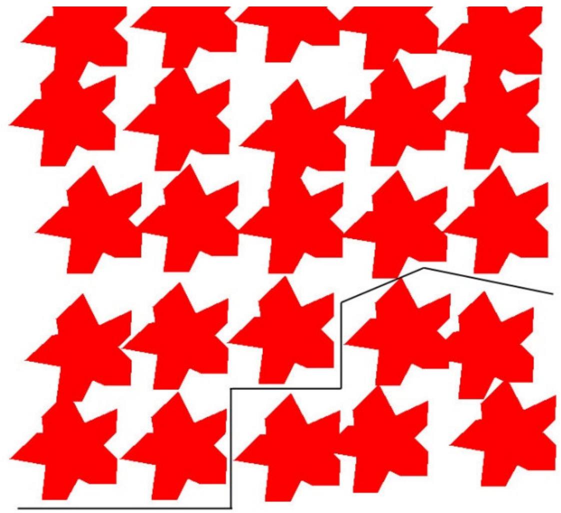
Applications of Mink sums, cont'd: Robot motion planing



robot, obstacles and computed path



computed planar map



Minkowski sum of the robot with obstacles

THE END