

Algorithms for 3D Printing and Other Manufacturing Methodologies

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Assembly Partitioning
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Outline

- 1 Assembly Partitioning
 - Polyhedral Assembly Partitioning with Infinite Translations

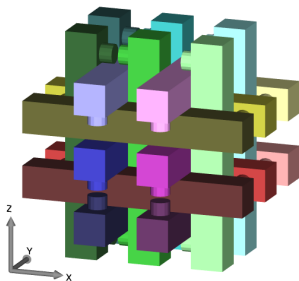


Outline

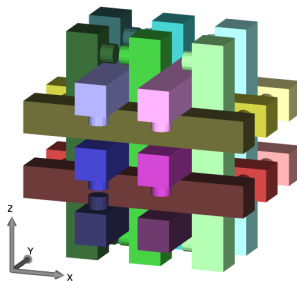
- 1 Assembly Partitioning
 - Polyhedral Assembly Partitioning with Infinite Translations



Assembly Properties



Interlocked



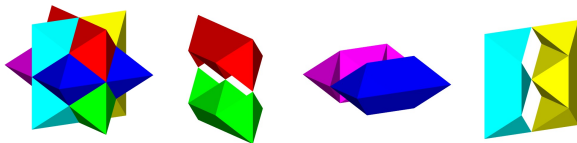
Separated



Polyhedral Assembly Partitioning with Infinite Translations

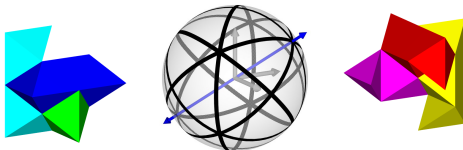
Input: n pairwise interior disjoint polytopes in \mathbb{R}^3

$$A = \{P_1, P_2, \dots, P_n\}$$



Output: A proper subset $S \subset A$ and a direction \vec{d} in \mathbb{R}^3 ,

- S can be translated as a rigid body to infinity along \vec{d} without colliding with $A \setminus S$
- Sliding motions of parts over other parts are allowed

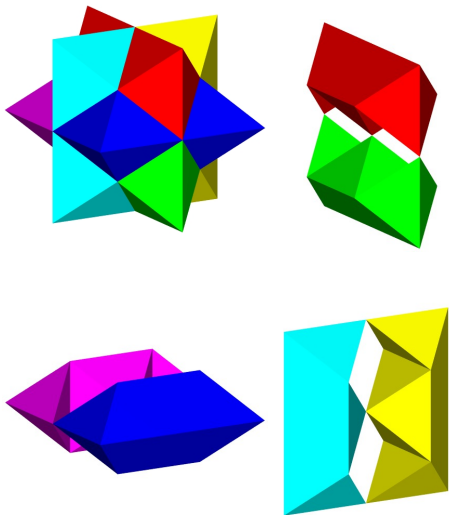


The Partitioning Process

- 1 Convex Decomposition
 - 2 Sub-part Gaussian map construction
 - 3 Sub-part Gaussian map reflection
 - 4 Pairwise sub-part Minkowski sum construction
 - 5 Pairwise sub-part Minkowski sum projection
 - 6 Pairwise Minkowski sum projection
 - 7 Motion-space construction
 - 8 Motion-space processing
- Different extensions of arrangements induced by geodesic arcs embedded on the sphere are used at different phases.
 - Only rational arithmetic is used.



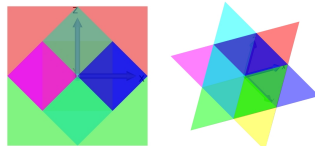
The Split Star Assembly



Is illustrated atop the right pedestal
in M. C. Escher's Waterfall woodcut



Viewed from different angles

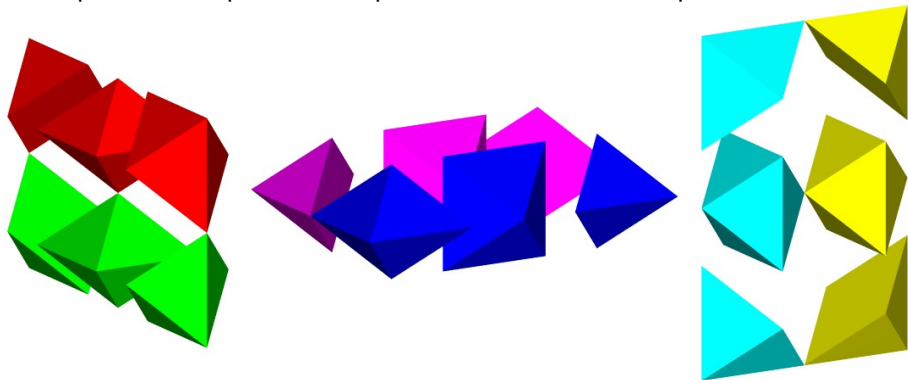


Has the shape of the first stellation of the rhombic dodecahedron



Phase 1: Convex Decomposition

The Split Star six parts decomposed into 3 convex sub-parts each



- We applied the decomposition operation manually
- We experimented with 3 different decompositions:
 - Each part decomposed into 3, 5, and 8 pieces
 - The 8-piece decomposition results with 48 identical tetrahedra



Phase 2: Sub-part Gaussian Map Construction

Samples of the Gaussian maps of sub-parts of the Split Star assembly



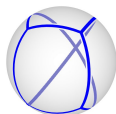
R_1



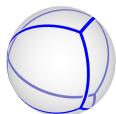
R_2



R_3



B_1



B_2



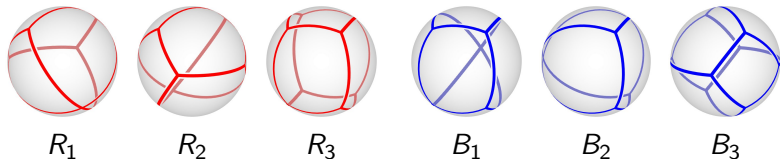
B_3

Output: Ordered list of parts, each part is an ordered list of the convex sub-part Gaussian-maps



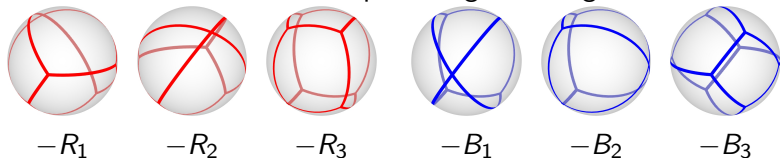
Phase 3: Sub-part Gaussian Map Reflection

Samples of the Gaussian maps of sub-parts of the Split Star assembly



Output: Ordered list of parts, each part is an ordered list of the convex sub-part Gaussian-maps

The reflections of the Gaussian-maps through the origin



Output: Ordered list of parts, each part is an ordered list of the convex sub-part Gaussian-maps



Phase 4: Pairwise Sub-part Minkowski Sum Construction

```
for  $i$  in  $\{1, 2, \dots, n\}$ 
  for  $j$  in  $\{1, 2, \dots, n\}$ 
    if  $i == j$  continue
    for  $k$  in  $\{1, 2, \dots, m_i\}$ 
      for  $\ell$  in  $\{1, 2, \dots, m_j\}$ 
         $M_{k\ell}^{ij} = P_{\ell}^j \oplus (-P_k^i)$ 
```

Output: A map from ordered pairs of distinct indices into lists of Minkowski sums



Phase 5: Pairwise Sub-part Minkowski Sum Projection

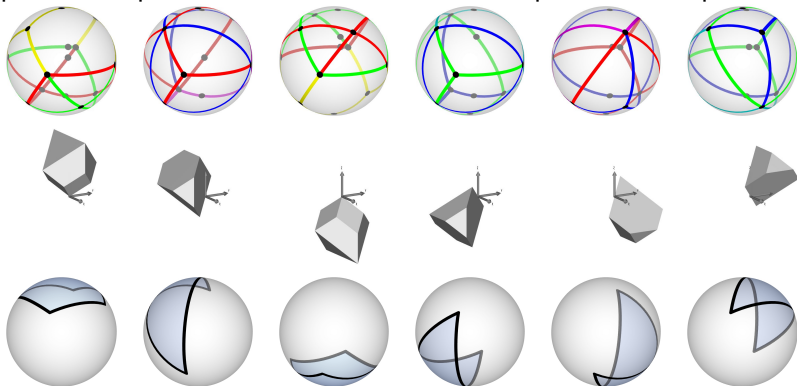
```
for  $i$  in  $\{1, 2, \dots, n\}$ 
  for  $j$  in  $\{1, 2, \dots, n\}$ 
    if  $i == j$  continue
    for  $k$  in  $\{1, 2, \dots, m_i\}$ 
      for  $\ell$  in  $\{1, 2, \dots, m_j\}$ 
         $Q_{k\ell}^{ij} = \text{project}(M_{k\ell}^{ij})$ 
```

Output: A map from ordered pairs of distinct indices into lists of central projections



Phase 5: Pairwise Sub-part Minkowski Sum Projection

Samples of the pairwise Minkowski sums of sub-parts of the Split Star



$R_1 \oplus (-G_1)$
 $R_1 \oplus (-B_1)$
 $G_1 \oplus (-R_1)$
 $G_1 \oplus (-B_1)$
 $B_1 \oplus (-R_1)$
 $B_1 \oplus (-G_1)$

- Middle row — Minkowski sums
- Top row — Gaussian maps
- Bottom row — central projection of the Minkowski sums on S^2



Phase 5: Pairwise Sub-part Minkowski Sum Projection

```
for  $i$  in  $\{1, 2, \dots, n\}$ 
  for  $j$  in  $\{1, 2, \dots, n\}$ 
    if  $i == j$  continue
    for  $k$  in  $\{1, 2, \dots, m_i\}$ 
      for  $\ell$  in  $\{1, 2, \dots, m_j\}$ 
         $Q_{k\ell}^{ij} = \text{project}(M_{k\ell}^{ij})$ 
```

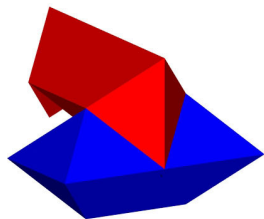
Given a convex Minkowski sum C , there are four different cases:

- 1 The origin is contained in the interior of a facet of C
- 2 The origin lies in the interior of an edge of C
- 3 The origin coincides with a vertex of C
- 4 The origin is separated from C

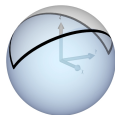


Phase 6: Pairwise Minkowski Sum Projection

```
for  $i$  in  $\{1, 2, \dots, n\}$ 
  for  $j$  in  $\{1, 2, \dots, n\}$ 
    if  $i == j$  continue
     $Q_{ij} = \emptyset$ 
    for  $k$  in  $\{1, 2, \dots, m_i\}$ 
      for  $\ell$  in  $\{1, 2, \dots, m_j\}$ 
         $Q_{ij} = Q_{ij} \cup Q_{kl}^{ij}$ 
```



$$B \oplus (-R)$$



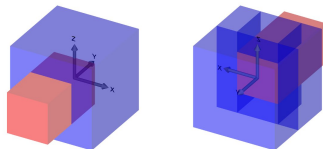
$$R \oplus (-B)$$

Output: A map from ordered pairs of distinct indices into single central projections



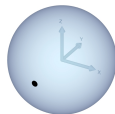
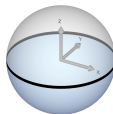
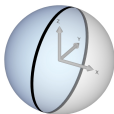
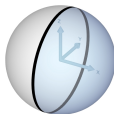
Phase 6: Pairwise Minkowski Sum Projection

The **Peg-in-the-hole** assembly viewed from two opposite directions

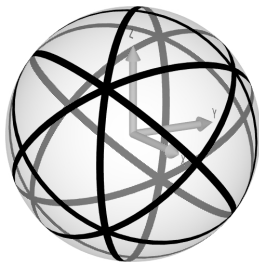


Sub-part Minkowski-sum projections

The union



Phase 7: Motion-Space Construction



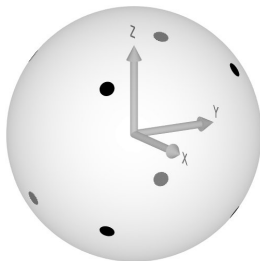
Output: A single arrangement that represents the **motion space**
Each arrangement cell is extended with a **directional blocking graph (DBG)**



Phase 8: Motion-Space Processing

- Traverse all vertices, edges, and faces of the **motion-space** arrangement
- Test the DBG associated with each cell for strong connectivity

	Direction	Subset
1.	-1,-1,-1	<i>GBT</i>
2.	-1,-1, 1	<i>RB T</i>
3.	-1, 1,-1	<i>GPT</i>
4.	-1, 1, 1	<i>RPT</i>
5.	1,-1,-1	<i>GBY</i>
6.	1,-1, 1	<i>RB Y</i>
7.	1, 1,-1	<i>GP Y</i>
8.	1, 1, 1	<i>RP Y</i>



Assembly Partitioning: Results

- Decompose each part into as few as possible sub-parts with as small as possible number of features
- An automatic decomposition operation that arrives at optimal or near optimal decompositions is expensive

Time consumption in seconds of the Split Star partitioning using different decompositions

A — number of convex sub-parts per part

B — number of sub-part vertices per part

C — total number of convex sub-parts

D — total number of Minkowski sums

E — total number of arrangements of geodesic arcs embedded on the sphere

A	B	C	D	E	1	2	3	4	5	6	7	8
3	16	18	270	607	NA	0.01	0.04	2.38	0.41	2.05		
5	22	30	750	1591	NA	0.01	0.05	5.03	1.09	7.07	0.36	0.01
8	32	48	1920	3967	NA	0.01	0.06	11.12	2.41	27.99		



Infinite Translational Partitioning in \mathbb{R}^3 : Complexity

- n —number of parts
- q —the maximum number of vertices per part.
- v —total number of vertices in the n parts

- Theoretical, exploiting similarity between DBGs associated with incident cells and avoiding convex decomposition
 - $O(v^2)$ —number of arcs inducing the motion-space arrangement
 - $O(v^4)$ —motion-space arrangement complexity
 - $O(v^4)$ —NDBG complexity
 - $O(n^{1.376})$ —strong connectivity computation amortized time consumption
 - $O(n^{1.376}v^4)$ —total time consumption

- Alternatively
 - $O(q^2)$ — Q_{ij} complexity
 - $O((nq)^4)$ —motion-space arrangement complexity

