Algorithms for 3D Printing and Other Manufacturing Methodologies

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CGAL
Apr. 3rd, 2017

Outline

1. CGAL
   - Introduction
   - Content
   - Literature
   - Details

Outline

CGAL: Mission

"Make the large body of geometric algorithms developed in the field of computational geometry available for industrial applications"

CGAL Project Proposal, 1996

CGAL Facts

- Written in C++
- Adheres the generic programming paradigm
- Development started in 1995
- Several active contributor sites
- High search-engine ranking for www.cgal.org

- Used in a diverse range of domains
  - e.g., computer graphics, scientific visualization, computer aided design and modeling, additive manufacturing, geographic information systems, molecular biology, medical imaging, and VLSI
- The de-facto standard in applied Computational Geometry

CGAL in Numbers

600,000	lines of C++ code
10,000	downloads per year not including Linux distributions
4,500	manual pages (user and reference manual)
1,000	subscribers to user mailing list
200	commercial users
120	packages
30	active developers
6	months release cycle
2	licenses: Open Source and commercial

CGAL History

Year	Version Released	Other Milestones
1996		CGAL founded
1998	July 1.1	
1999		Work continued after end of European support
2001	Aug 2.3	Editorial Board established
2002	May 2.4	
2003	Nov 3.0	Geometry Factory founded
2004	Dec 3.1	
2005		
2006	May 3.2	
2007	Jun 3.3	
2008		CMake
2009	Jan 3.4, Oct 3.5	
2010	Mar 3.6, Oct 3.7	Google Summer of Code (GSoC) 2010
2011	Apr 3.8, Aug 3.9	GSoC 2011
2012	Mar 4.0, Oct 4.1	GSoC 2012
2013	Mar 4.2, Oct 4.3	GSoC 2013, Doxygen
2014	Apr 4.4, Oct 4.5	GSoC 2014
2015	Apr 4.6, Oct 4.7	GitHub, HTML5, Main repository made public
2016	Apr 4.8, Sep 4.9	20^{th} anniversary
2017		GSoC 2017

CGAL Properties

- Reliability
  - Explicitly handles degeneracies
  - Follows the Exact Geometric Computation (EGC) paradigm
- Efficiency
  - Depends on leading 3rd party libraries
    - e.g., Boost, Gmp, Mpfr, Qt, Eigen, Tbb, and Core
  - Adheres to the generic-programming paradigm
    - Polymorphism is resolved at compile time

⇒ The best of both worlds ⇐

CGAL Properties, Cont

- Flexibility
  - Adaptable, e.g., graph algorithms can directly be applied to CGAL data structures
  - Extensible, e.g., data structures can be extended
- Ease of Use
  - Has didactic and exhaustive Manuals
  - Follows standard concepts (e.g., C++ and STL)
  - Has a modular structure, e.g., geometry and topology are separated
  - Characterizes with a smooth learning-curve

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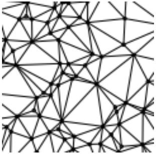
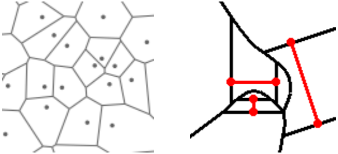
2D Algorithms and Data Structures



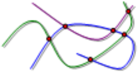
Triangulations

Mesh Generation

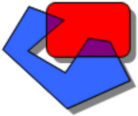
Polyline Simplification



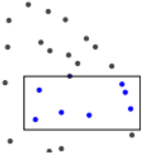
Voronoi Diagrams



Arrangements

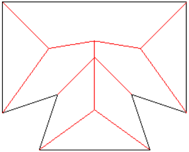


Boolean Operations



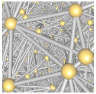
Neighborhood Queries

Minkowski Sums

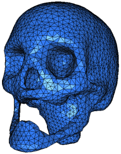


Straight Skeleton

3D Algorithms and Data Structures

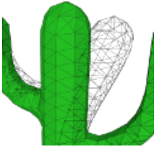


Triangulations

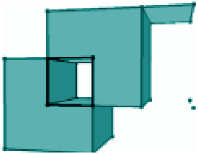


Mesh Generation

Polyhedral Surface

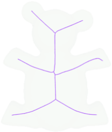


Deformation



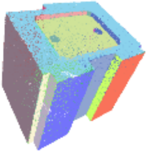
Boolean Operations

Mesh Simplification

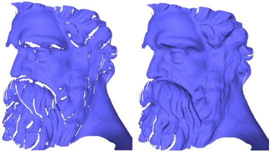


Skeleton

Segmentation



Classification



Hole Filling

Outline

CGAL Bibliography I

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  A generic lazy evaluation scheme for exact geometric computations.
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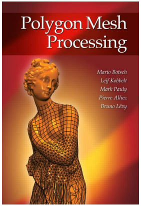
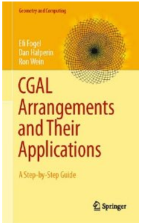
- M. H. Overmars.
  Designing the computational geometry algorithms library CGAL.
  In Proceedings of ACM Workshop on Applied Computational Geometry, Towards Geometric Engineering, volume 1148, pages 53–58, London, UK, 1996. Springer.

- Many Many Many papers





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CGAL Structure

Basic Library

Algorithms and Data Structures

e.g., Triangulations, Surfaces, and Arrangements

Kernel

Elementary geometric objects

Elementary geometric computations on them

Visualization

Files

I/O

Number Types

Generators

Support Library

Configurations, Assertions,...

CGAL Basic Library

- Generic data structures are parameterized with Traits
  - Separates algorithms and data structures from the geometric kernel.
- Generic algorithms are parameterized with iterator ranges
  - Decouples the algorithm from the data structure.

CGAL Components Developed at Tel Aviv University

- 2D Arrangements
- 2D Envelopes
- Inscribed Areas / 2D Largest empty iso rectangle

CGAL Components Developed at Tel Aviv University

- 2D Arrangements
- 2D Regularized Boolean Set-Operations
- 2D Minkowski Sums
- 2D Envelopes
- 3D Envelopes
- 2D Snap Rounding
- Inscribed Areas / 2D Largest empty iso rectangle

CGAL 2D Arrangements

- The main data structure

```
template <typename Traits, typename Dcel> Arrangement_2 { ... };
```

Traits definitions of geometric elements

- geometric-object types e.g., Point_2, and
- operations on objects of these types, e.g., Compare_xy_2.

Dcel definitions of topological elements

- topological-object types, e.g., vertex, halfedge, and face, and
- operations required to maintain the incidence relations among objects of these types.

- A traits class for line segments.

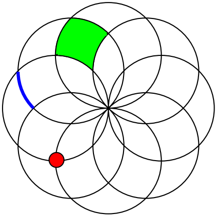
```
Arr_non_cached_segment_traits_2<Kernel> : public Kernel { ... };
```

- All object types and most operations are inherited from the derived kernel.

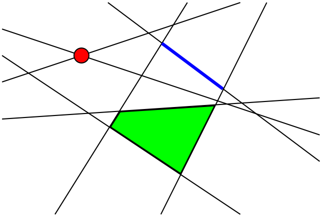
Two Dimensional Arrangements

Definition (Arrangement)

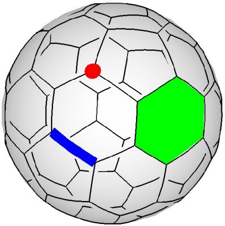
Given a collection $\mathscr{C}$ of curves on a surface, the arrangement $\mathscr{A}(\mathscr{C})$ is the partition of the surface into vertices, edges and faces induced by the curves of $\mathscr{C}$.



An arrangement of circles in the plane.



An arrangement of lines in the plane.



An arrangement of great-circle arcs on a sphere.

CGAL Kernel Concept

- Geometric objects of constant size.
- Geometric operations on object of constant size.

Primitives 2D, 3D, dD		Operations	
		Predicates	Constructions
point	●	comparison	intersection
vector	→	orientation	squared distance
triangle	△	containment	. . .
iso rectangle	▭	. . .	
circle	○		
. . .			

CGAL Kernel Affine Geometry

point - origin	$\rightarrow$ vector
point - point	$\rightarrow$ vector
point + vector	$\rightarrow$ point

point + point	$\leftarrow$ Illegal

midpoint$(a, b) = a + 1/2 \times (b - a)$

CGAL Kernel Classification

- Dimension: 2, 3, arbitrary
- Number types:
  - Ring: $+, -, \times$
  - Euclidean ring (adds integer division and gcd) (e.g., CGAL::Gmpz).
  - Field: $+, -, \times, /$ (e.g., CGAL::Gmpq).
  - Exact sign evaluation for expressions with roots (Field_with_sqr).
- Coordinate representation
  - Cartesian—requires a field number type or Euclidean ring if no constructions are performed.
  - Homegeneous—requires Euclidean ring.
- Reference counting
- Exact, Filtered

CGAL Kernels and Number Types

Cartesian representation	Homogeneous representation
point $\left\lvert \begin{array}{l} x = \frac{hx}{hw} \\ y = \frac{hy}{hw} \end{array}\right.$	point $\left\lvert \begin{array}{l} hx \\ hy \\ hw \end{array}\right.$

Intersection of two lines

$$\begin{cases} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{cases} \qquad \begin{cases} a_1hx + b_1hy + c_1hw = 0 \\ a_2hx + b_2hy + c_2hw = 0 \end{cases}$$

$$(x, y) = \left(\frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, -\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \right)$$

Field operations

$$(hx, hy, hw) = \left(\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, -\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right)$$

Ring operations

Example: Kernels<NumberType>

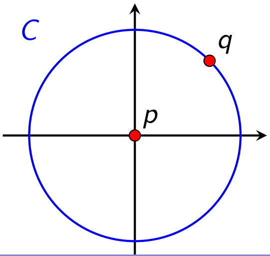
- Cartesian<FieldNumberType>
  - `typedef CGAL::Cartesian<Gmpq> Kernel;`
  - `typedef CGAL::Simple_cartesian<double> Kernel;`
    - ⋆ No reference-counting, inexact instantiation
- Homogeneous<RingNumberType>
  - `typdef CGAL::Homogeneous<Core::BigInt> Kernel;`
- d-dimensional `Cartesian_d` and `Homogeneous_d`
- Types + Operations
  - `Kernel::Point_2`, `Kernel::Segment_3`
  - `Kernel::Less_xy_2`, `Kernel::Construct_bisector_3`

CGAL Numerical Issues

```
#if 1
  typedef CORE::Expr                              NT;
  typedef CGAL::Cartesian<NT>                     Kernel;
  NT sqrt2 = CGAL::sqrt(NT(2));
#else
  typedef double                                  NT;
  typedef CGAL::Cartesian<NT>                     Kernel;
  NT sqrt2 = sqrt(2);
#endif

Kernel::Point_2 p(0,0), q(sqrt2,sqrt2);
Kernel::Circle_2 C(p,4);
assert(C.has_on_boundary(q));
```

- OK if NT supports exact sqrt.
- Assertion violation otherwise.



CGAL Pre-defined Cartesian Kernels

- Support construction of points from `double` Cartesian coordinates.
- Support exact geometric predicates.
- Handle geometric constructions differently:
  - `CGAL::Exact_predicates_inexact_constructions_kernel`
    - ⋆ Geometric constructions may be inexact due to round-off errors.
    - ⋆ It is however more efficient and sufficient for most CGAL algorithms.
  - `CGAL::Exact_predicates_exact_constructions_kernel`
  - `CGAL::Exact_predicates_exact_constructions_kernel_with_sqrt`
    - ⋆ Its number type supports the exact square-root operation.

CGAL Special Kernels

- Filtered kernels
- 2D circular kernel
- 3D spherical kernel

- Refer to CGAL's manual for more details.

Computing the Orientation

- imperative style

```
#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel  Kernel;
typedef Kernel::Point_2                                       Point_2;

int main()
{
  Point_2 p(0,0), q(10,3), r(12,19);
  return (CGAL::orientation(q,p,r) == CGAL::LEFT_TURN) ? 0 : 1;
}
```

- precative style

```
#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel  Kernel;
typedef Kernel::Point_2                                       Point_2;
typedef Kernel::Orientation_2                                 Orientation_2;

int main()
{
  Kernel kernel;
  Orientation_2 orientation = kernel.orientation_2_object();

  Point_2 p(0,0), q(10,3), r(12,19);
  return (orientation(q,p,r) == CGAL::LEFT_TURN) ? 0 : 1;
}
```

CGAL Adaptable & Extensible Kernel

Geometric class templates are parameterized with the kernel

```
template <typename K> struct MyPoint { ... };
template <typename K> struct MyLine { ... };
template <typename K> struct MyConstruct { ... };
```

Geometric class definitions are nested in the kernel

```
struct Kernel {
  typedef MyPoint<Kernel>        Point_2;
  typedef MyLine<Kernel>         Line_2;
  typedef MyConstruct<Kernel>    Construct_line_2;
};
```

Injecting a class into its nested templates is not a problem, but there is more...

CGAL Adaptable & Extensible Kernel, Cont

We want to define a new kernel where new types can be added and existing ones can be exchanged

```
struct New_kernel : public Kernel {
  typedef NewPoint<New_kernel>    Point_2;
  typedef MyLeftTurn<New_kernel> Left_turn_2;
};
```

- Problem: The inherited class `MyConstruct` is still parameterized with Kernel, hence it operates on the old point class `MyPoint`.
- Solutions
  - Redefine `Construct_line_2` in `New_kernel`

CGAL Adaptable & Extensible Kernel, Cont

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CGAL Adaptable & Extensible Kernel, Cont

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};
```

- Problem: The inherited class `MyConstruct` is still parameterized with Kernel, hence it operates on the old point class `MyPoint`.
- Solutions
  - ~~Redefine `Construct_line_2` in `New_kernel`~~
  - Defer the instantiation of `Construct_line_2`

CGAL Adaptable & Extensible Kernel, Cont

```
template <typename K>
struct Kernel_base {
  typedef MyPoint<K>        Point_2;
  typedef MyLine<K>         Line_2;
  typedef MyConstruct<K>    Construct_line_2;
};
struct Kernel : public Kernel_base<Kernel> {};
```

Defer instantiation once again to be able to extend New_kernel in the same way as Kernel.

```
template <typename K>
struct New_kernel_base : public Kernel_base<K> {
  typedef NewPoint<K>       Point_2;
  typedef MyLeftTurn<K>     Left_turn_2;
};
struct New_kernel : public New_kernel_base<New_kernel> {};
```

Computing the Intersection

```
#include <CGAL/Exact_predicates_inexact_constructions_kernel.h>
#include <CGAL/intersections.h>

typedef CGAL::Exact_predicates_inexact_constructions_kernel Kernel;
typedef Kernel::Point_2   Point_2;
typedef Kernel::Segment_2 Segment_2;
typedef Kernel::Line_2    Line_2;

int main() {
  Point_2 p(1,1), q(2,3), r(-12,19);
  Line_2 line(p,q);
  Segment_2 seg(r,p);
  auto result = CGAL::intersection(seg, line);
  if (result) {
    if (const Segment_2* s = boost::get<Segment_2>(&*result)) {
      // handle segment
    }
    else {
      const Point_2* p = boost::get<Point_2>(&*result);
      // handle point
    }
  }
  return 0;
}
```