

Assignment no. 2

due: Wednesday, March 30th, 2022

Exercise 2.1 Give an efficient algorithm to determine whether a simple polygon with n vertices is monotone with respect to some given line, not necessarily a vertical or horizontal one. Analyze the running time of your algorithm.

Exercise 2.2 The *stabbing number* of a triangulation of a simple polygon P is the maximum number of diagonals intersected by any line segment interior to P . Give an algorithm that computes a triangulation of a convex polygon that has stabbing number $O(\log n)$.

Exercise 2.3 Prove that the following polyhedron $\mathcal{P}$ cannot be tetrahedralized using only vertices of $\mathcal{P}$, namely its interior cannot be partitioned into tetrahedra whose vertices are selected from the vertices of $\mathcal{P}$ (see the enclosed figure).¹

Let a, b, c be the vertices (labeled counterclockwise) of an equilateral triangle in the xy -plane. Let a', b', c' be the vertices of abc when translated up to the plane $z = 1$. Define an intermediate polyhedron $\mathcal{P}'$ as the hull of the two triangles including the diagonal edges ab', bc' , and ca' , as well as the vertical edges aa', bb' , and cc' , and the edges of the two triangles abc and $a'b'c'$. Now twist the top triangle $a'b'c'$ by 30° in the plane $z = 1$, rotating and stretching the attached edges accordingly: this is the polyhedron $\mathcal{P}$.

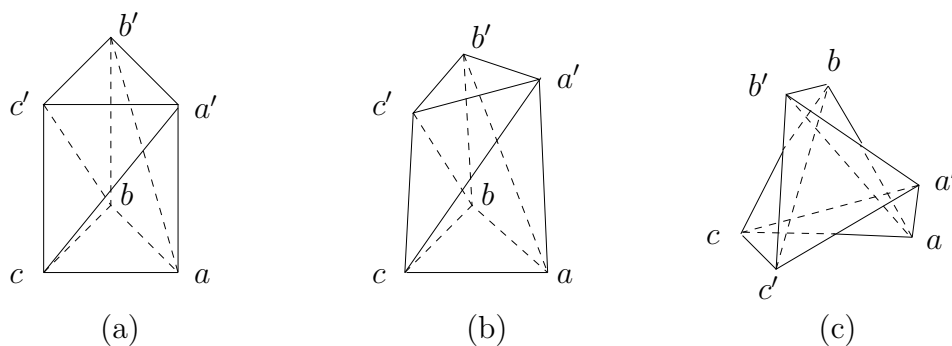


Figure 1: The untetrahedralizable polyhedron is constructed by twisting the top of a triangular prism (a) by 30° degrees, producing (b), shown in top view in (c)

Notice that there are additional exercises on the other side of the page.

¹This construction is due to Schönhardt, 1928. The description here is taken from O'Rourke's *Art Gallery Theorems and Algorithms*.

Exercise 2.4 In each of the following settings, describe a construction (a polygon and guard placements) where the specified **vertex guards** do not fully cover the polygon in the art gallery sense, and such that your construction could be generalized to any number of vertices, as specified in the setting:

- (a) A simple polygon with $2k$ vertices, for every $k > 2$, and a specific assignment of guards placed at **every other vertex** along the boundary of the polygon. Namely, guards placed at the vertices $v_i, v_{i+2}, v_{i+4}, \dots$, do not fully cover the polygon.
- (b) Similarly, a simple polygon with $3k$ vertices, for every $k > 2$, and a specific assignment of guards placed at **every third vertex** along the boundary of the polygon.
- (c) A simple polygon with n vertices, for every $n > 5$, and guards placed only at **convex vertices**. A vertex is convex if its interior angle is less than π .

Exercise 2.5 (Optional, bonus) The *pockets* of a simple polygon are the areas outside the polygon, but inside its convex hull. Let P_1 be a simple polygon with n_1 vertices, and assume that a triangulation of P_1 as well as of its pockets is given. Let P_2 be a convex polygon with n_2 vertices. Show that the intersection $P_1 \cap P_2$ can be computed in $O(n_1 + n_2)$ time. (CGAA Ex. 3.12)

Exercise 2.6 (Self learning, do not submit) The first part (a) is in preparation for the next topic.

Let $f_i(x), i = 1, \dots, n$ be a set of functions. The lower envelope Ψ of this set of functions is the pointwise minimum of these functions: $\Psi(x) = \min_i f_i(x)$.

(a) Assume that the functions are linear, namely $f_i(x) = a_i x + b_i$. We divide the x -axis into maximal interval such that for each interval the minimum is attained by the same function f_i .

(a1) What is the maximum number of such maximal intervals for any collection of linear functions.

(a2) Describe a divide-and conquer algorithm to efficiently compute the lower envelope of a set of linear functions. Analyze the time and space required by the algorithm.

(b, harder) The same as (a), only this time the functions are parabolas, namely $f_i(x) = a_i x^2 + b_i x + c_i$.