More on
Arrangements and Duality

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Overview

- 3-points-on-a-line is 3SUM-hard
- Minimum area triangle
- Sum of squared face complexity
- Maximum level vertex
- Non-linear objects

Credits

- CGAA: Chapter 8 in Computational Geometry Algorithms and Applications, Arrangements and Duality, by de Berg et al
- L11vK: Lecture 11 in Computational Geometry, Arrangements and Duality, by Marc van Kreveld

3-points-on-a-line is 3SUM-hard

3SUM

- Given a set S of n integers, are there three elements $a, b, c \in S$ such that $a + b + c = 0$

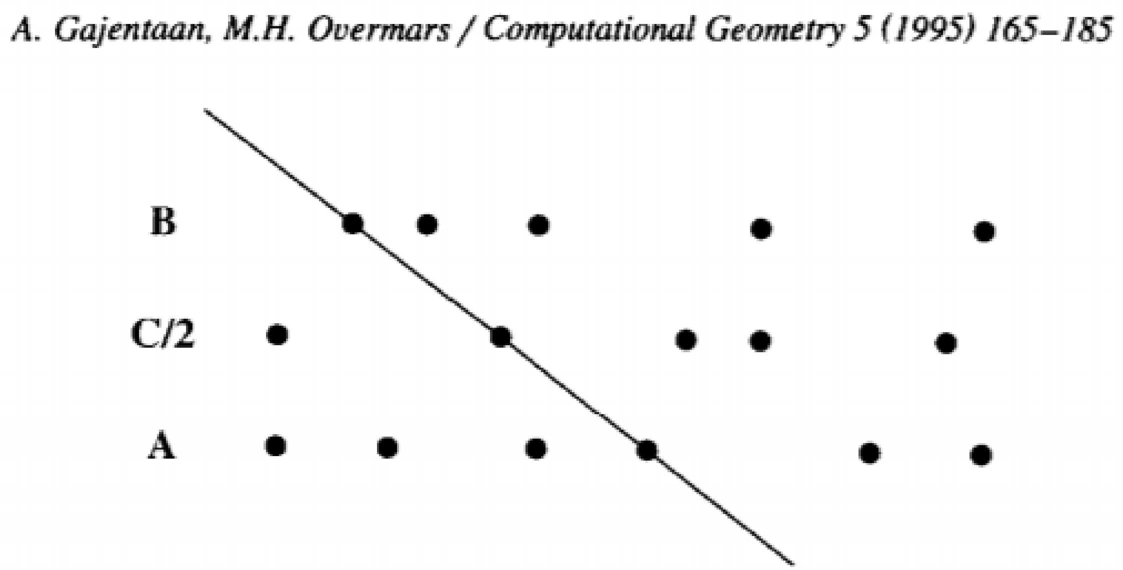
3SUM-hardness

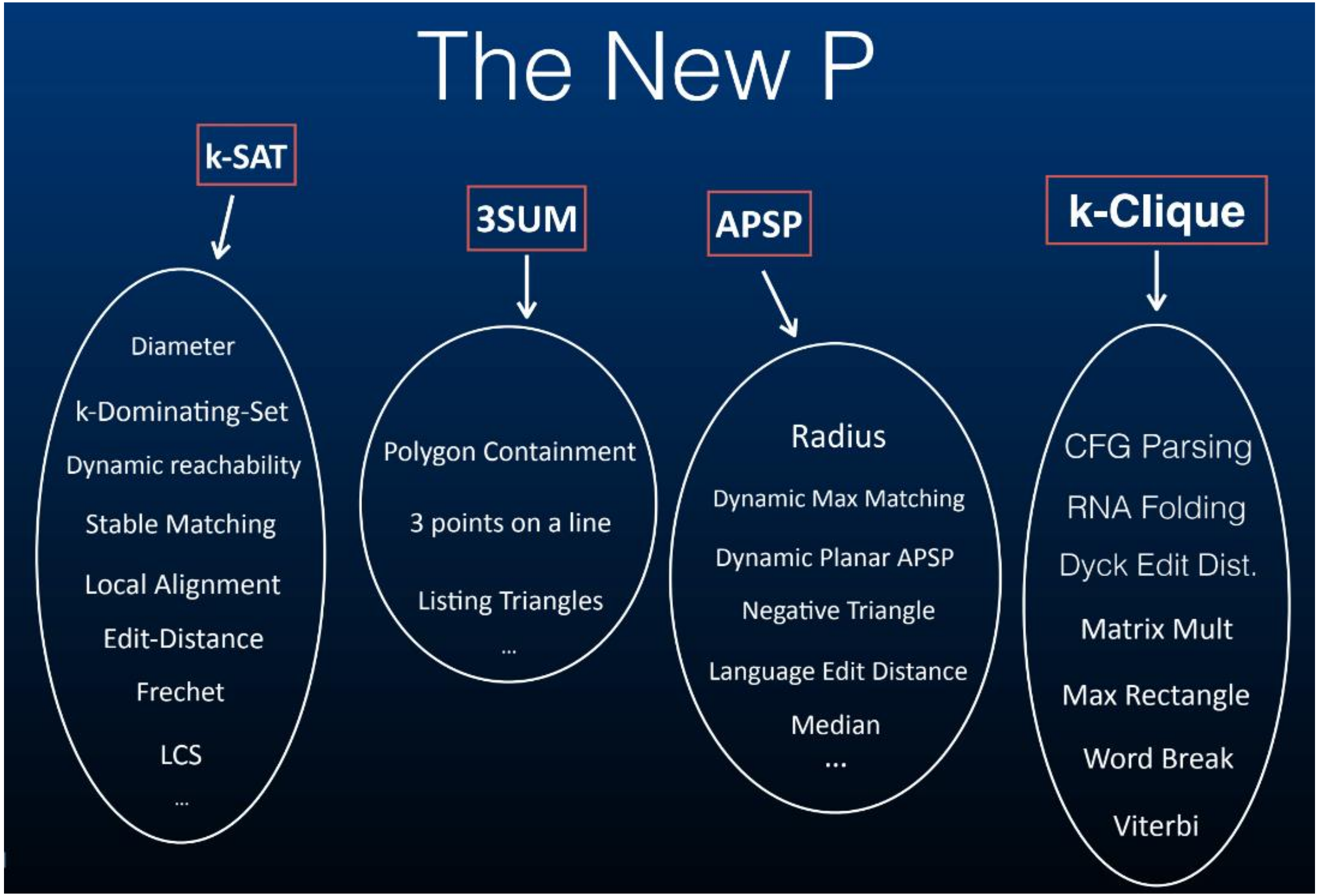
- [Gajentaan & Overmars ‘1995]
- A problem is 3SUM-hard if solving it in subquadratic time implies a subquadratic-time algorithm for 3SUM
- 3-points-on-a-line is 3SUM-hard
- 3SUM is by now solvable by a subquadratic-time algorithm [Grønlund-Pettie ‘14] and further developments
- No subquadratic-time solution is known for 3-points-on-a-line

3-points-on-a-line is 3SUM-hard

- [Gajentaan & Overmars '1995]
- 3SUM': Given three sets of integers A,B,C of total size n, are there elements $a \in A, b \in B, c \in C$ such that $a + b = c$
- 3SUM and 3SUM' are linear-time transformable to one another
- 3-points-on-a-line is 3SUM'-hard
- A is on $y = 0$, B on $y = 2$, and

$C/2$ on $y = 1$




The New P
k-SAT
Diameter
k-Dominating-Set
Dynamic reachability
Stable Matching
Local Alignment
Edit-Distance
Frechet
LCS
...
3SUM
Polygon Containment
3 points on a line
Listing Triangles
...
APSP
Radius
Dynamic Max Matching
Dynamic Planar APSP
Negative Triangle
Language Edit Distance
Median
...
k-Clique
CFG Parsing
RNA Folding
Dyck Edit Dist.
Matrix Mult
Max Rectangle
Word Break
Viterbi

[Abboud 2016]

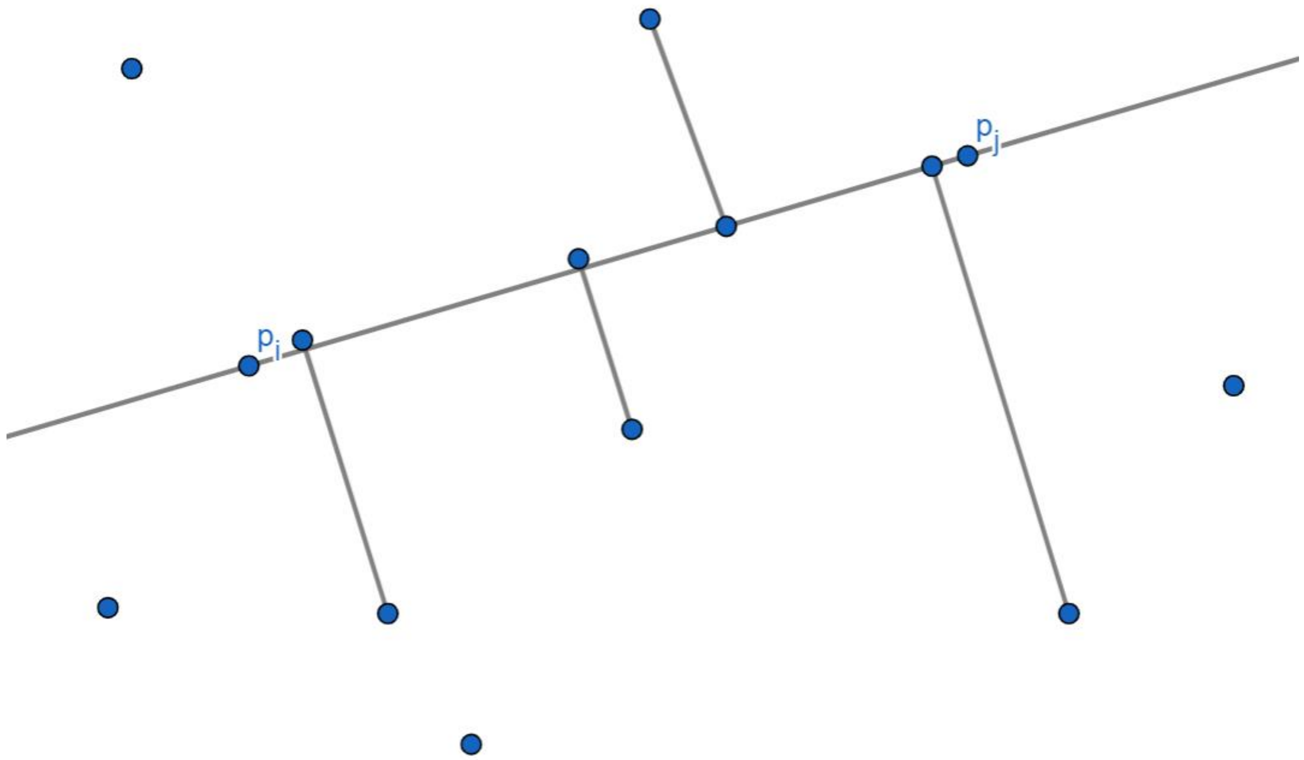
The minimum area triangle

Problem: Minimum area triangle

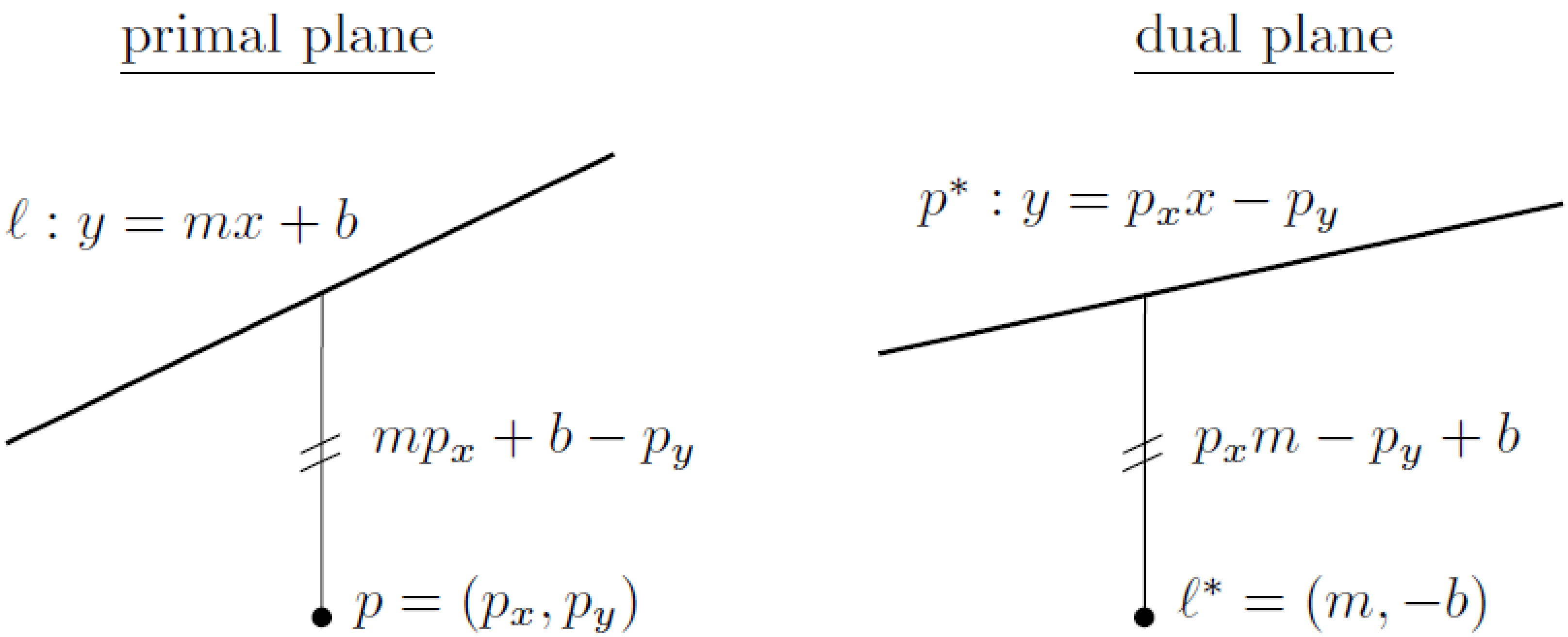
- Given a set of n points $p_1, \dots, p_n$ in the plane find the three that define the triangle of minimum area
- The problem is 3SUM-hard

Subproblem for p_i, p_j

- Find a third point p_k such that among all other points defines with p_i, p_j the smallest area triangle



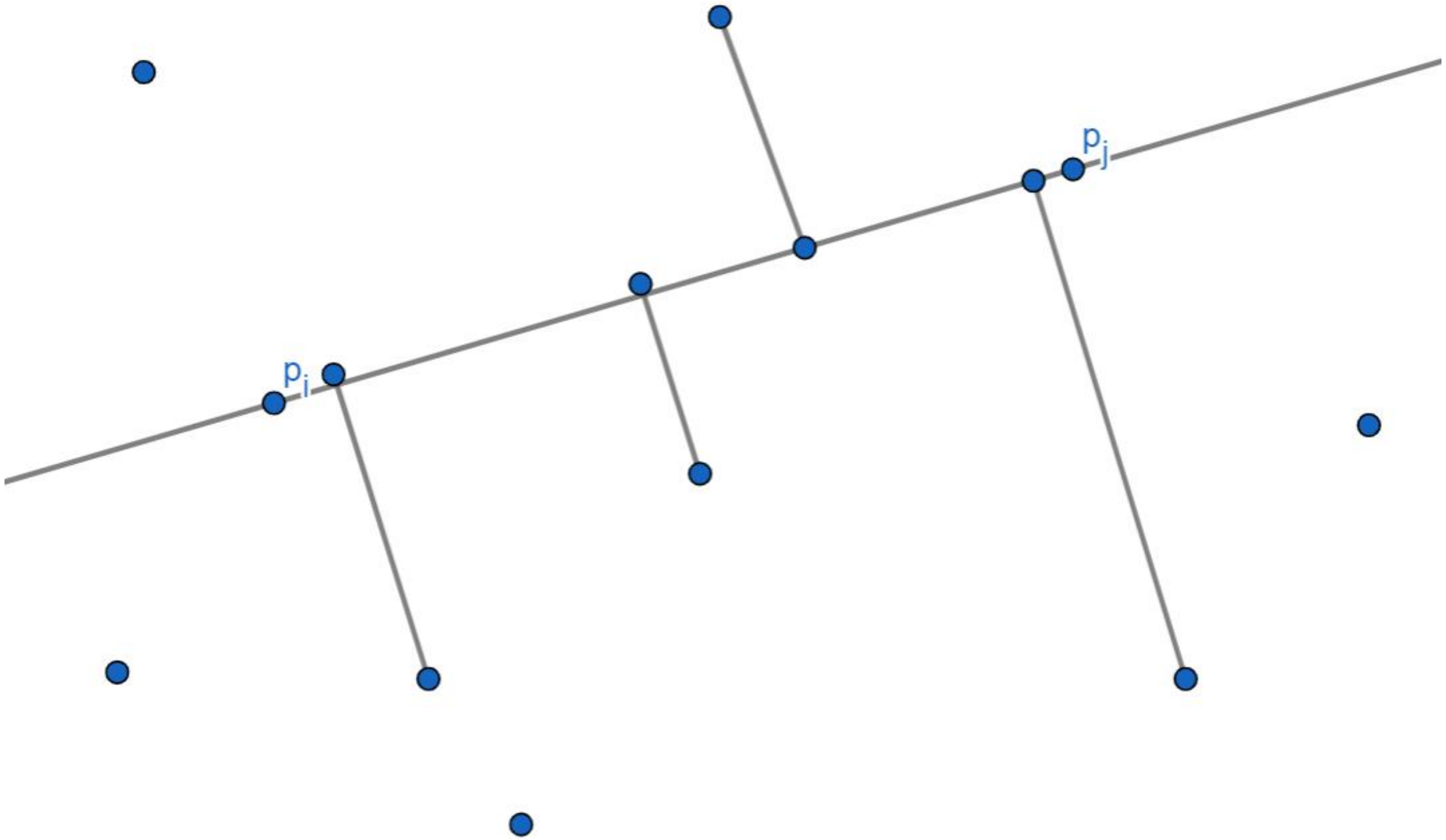
Recall



Duality preserves vertical distances

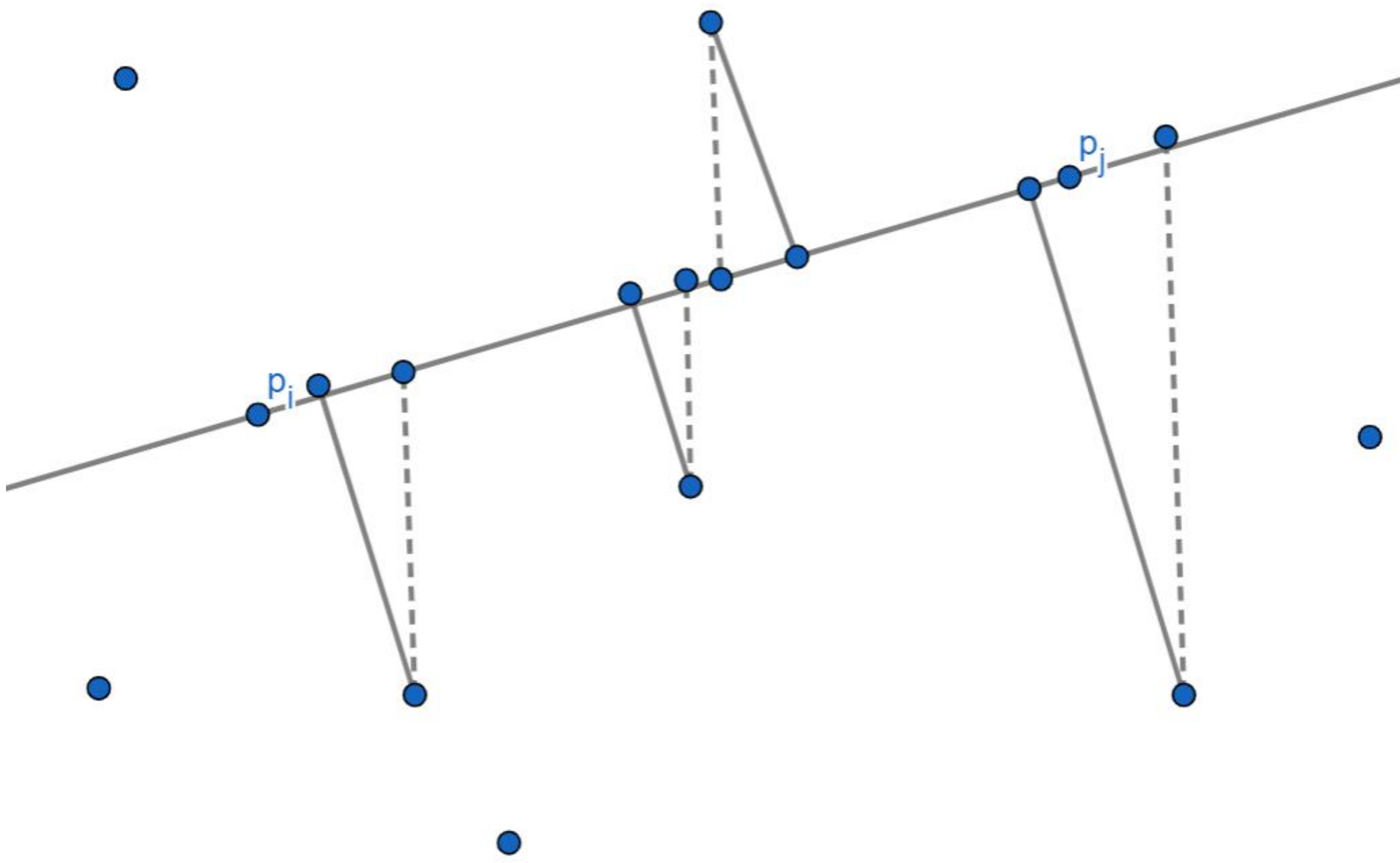
[L11vK]

?



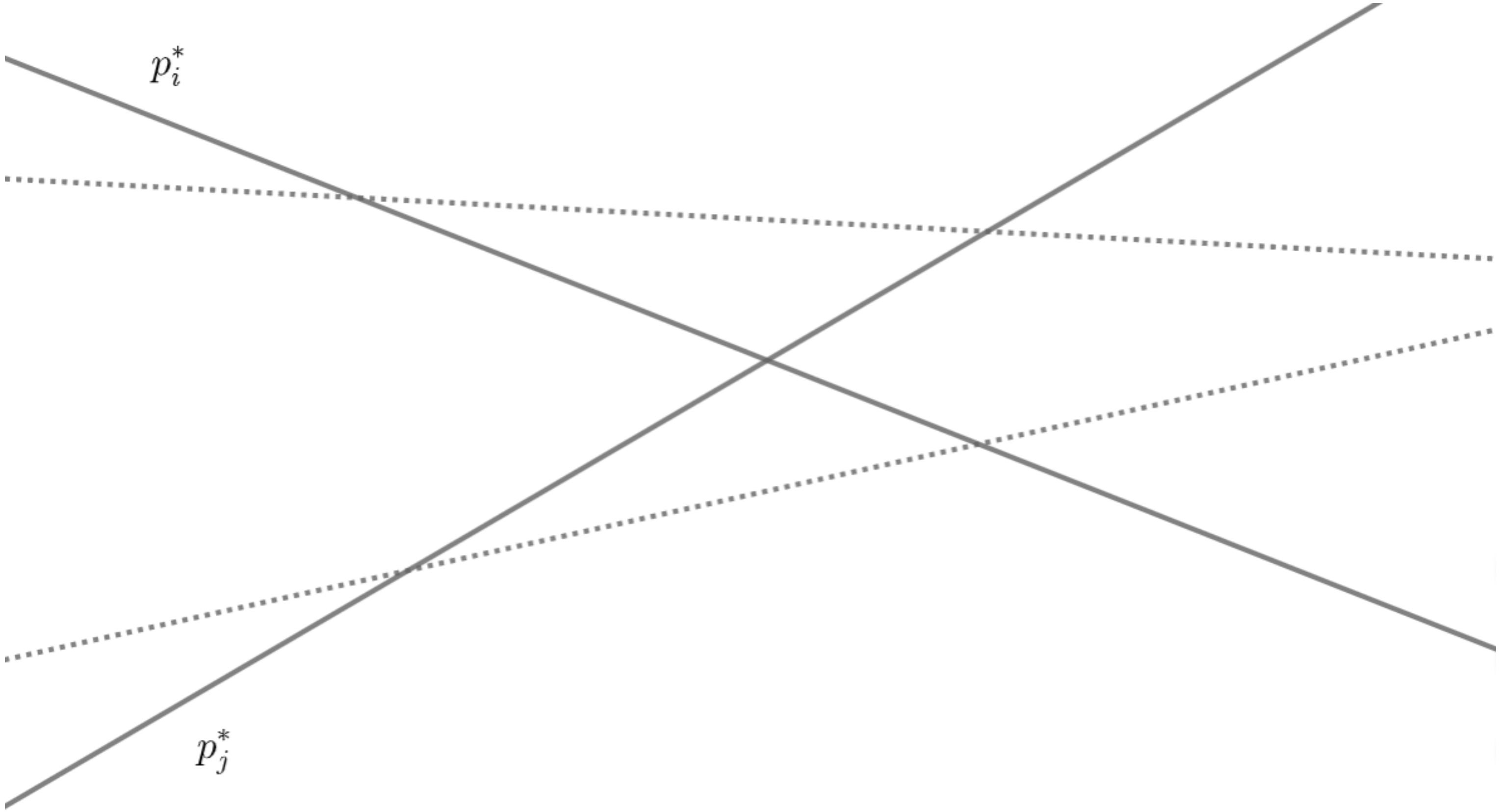
p_i
p_j

!



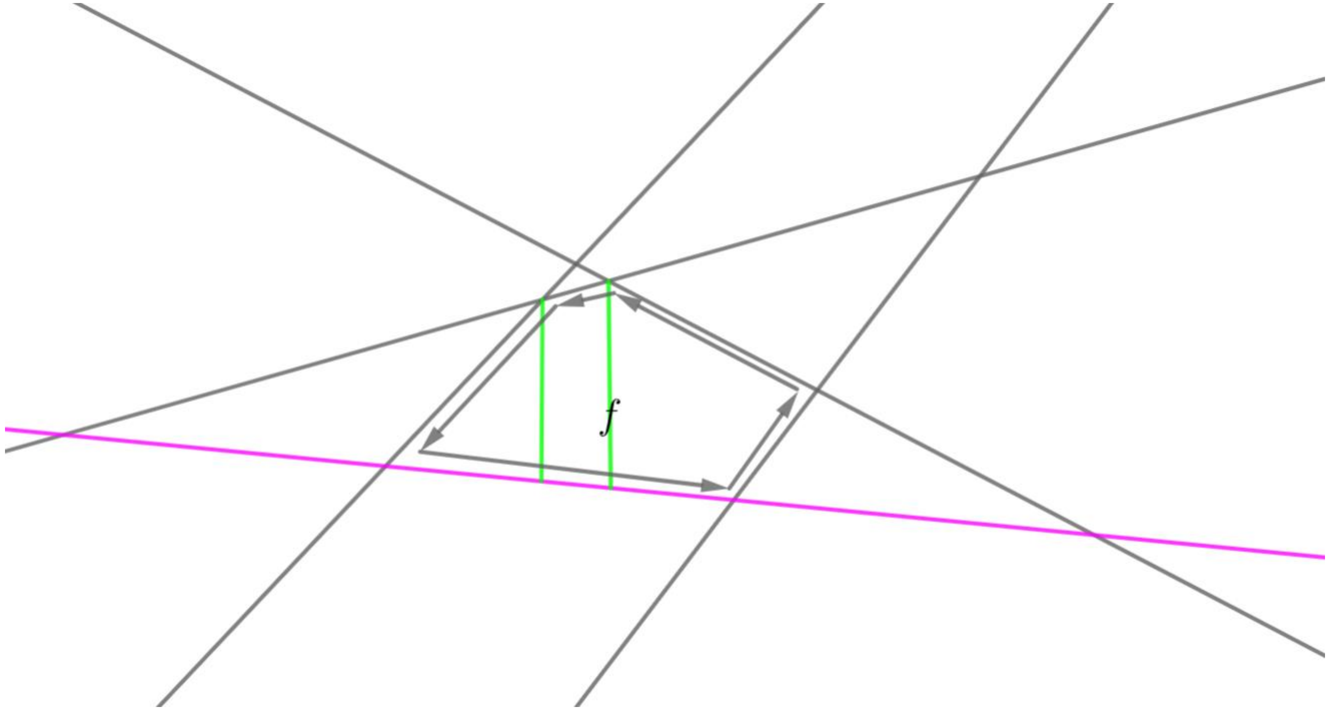
p_i
p_j

How does this look in the dual plane?



The traversal

- Line by line ($p_k{}^*$)
- Traversing the zone of the line $p_k{}^*$, looking for candidate $p_i{}^* \cap p_j{}^*$



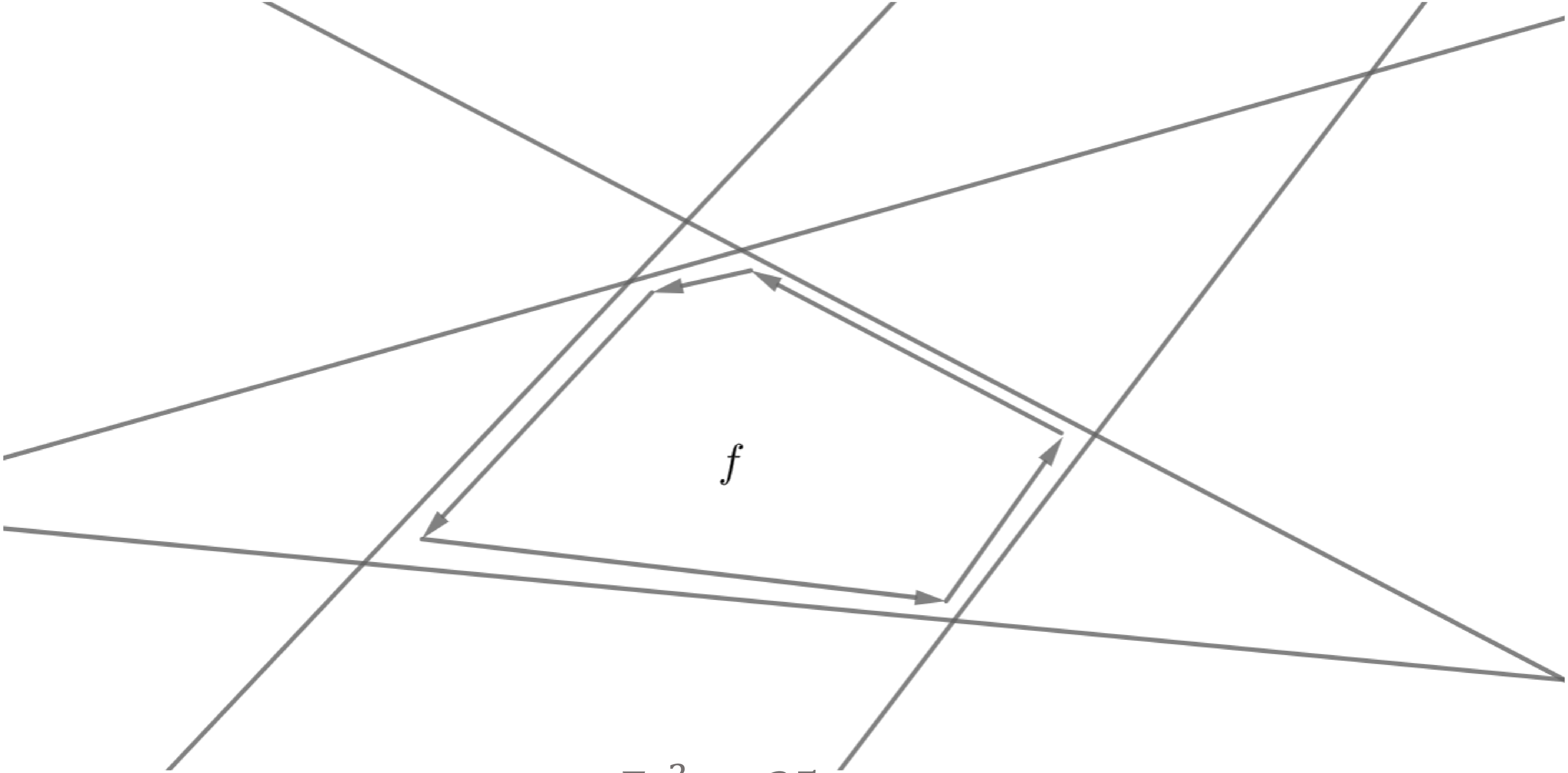
Remarks

- Running time $O(n^2)$
- The question is relevant in any dimension: minimum volume simplex
- Runs in $O(n^d)$ time in R^d, assuming d is fixed
- Relies on a corresponding zone theorem in higher dimension

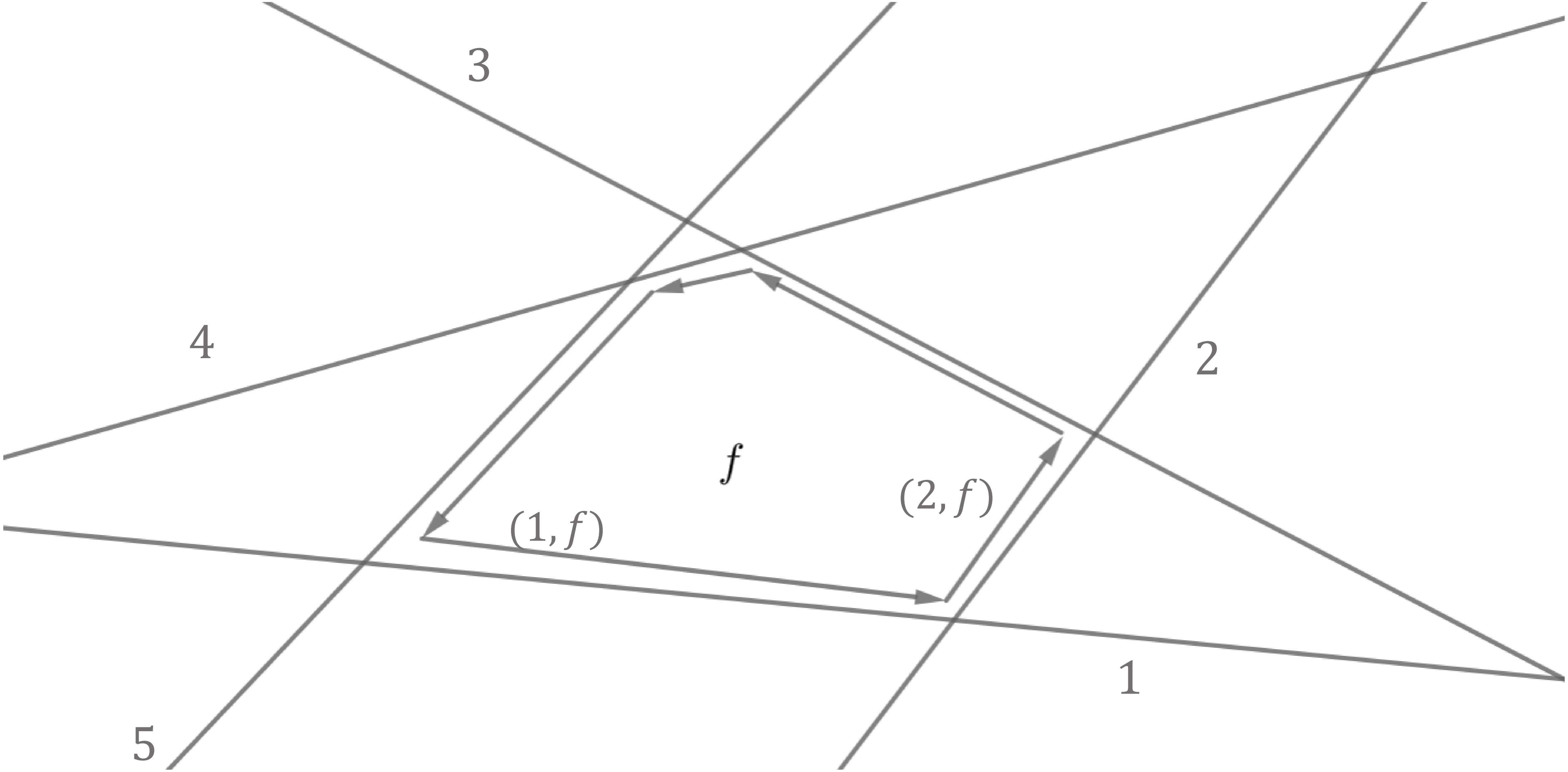
Sum of squared face complexity

The problem

- Given an arrangement of n lines in the plane.
- Let E_f be the number of edges along the boundary of face f (including unbounded faces and edges).
- What is the maximum value of $\sum E_f{}^2$, where the sum is over all faces of the arrangement?



$$E_f{}^2 = 25$$



3
4
2
f
$(2, f)$
$(1, f)$
1
5

	$(1, f)$	$(2, f)$	$(3, f)$	$(4, f)$	$(5, f)$
$(1, f)$					
$(2, f)$					
$(3, f)$					
$(4, f)$					
$(5, f)$					

	$(1, f)$	$(2, f)$	$(3, f)$	$(4, f)$	$(5, f)$
$(1, f)$					
$(2, f)$					
$(3, f)$					
$(4, f)$					
$(5, f)$					

the contribution of face f to the zone of line 1

Finding the max-level vertex

The problem

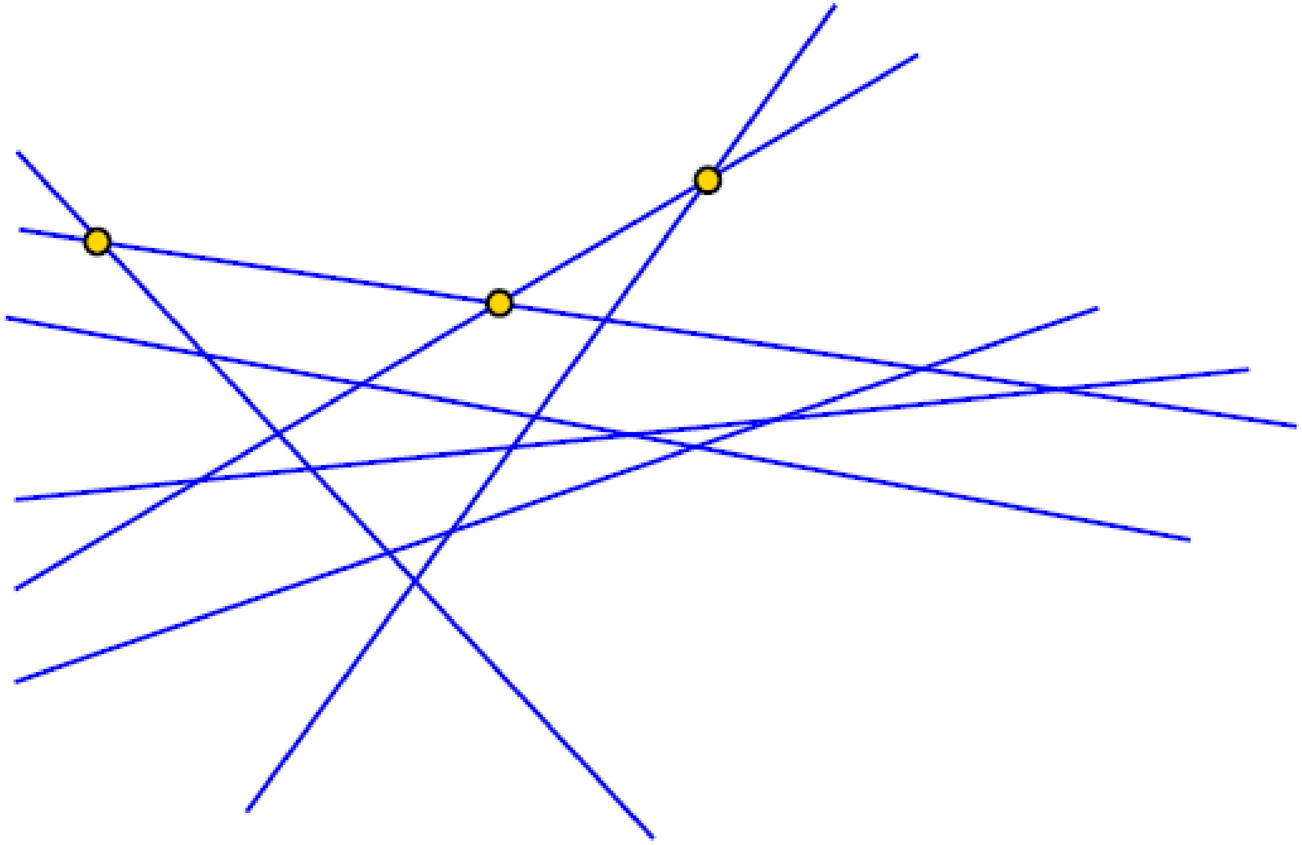
Chapter 8
ARRANGEMENTS AND DUALITY

8.13 Given a set L of n lines in the plane, give an $O(n \log n)$ time algorithm to compute the maximum level of any vertex in the arrangement $\mathcal{A}(L)$.

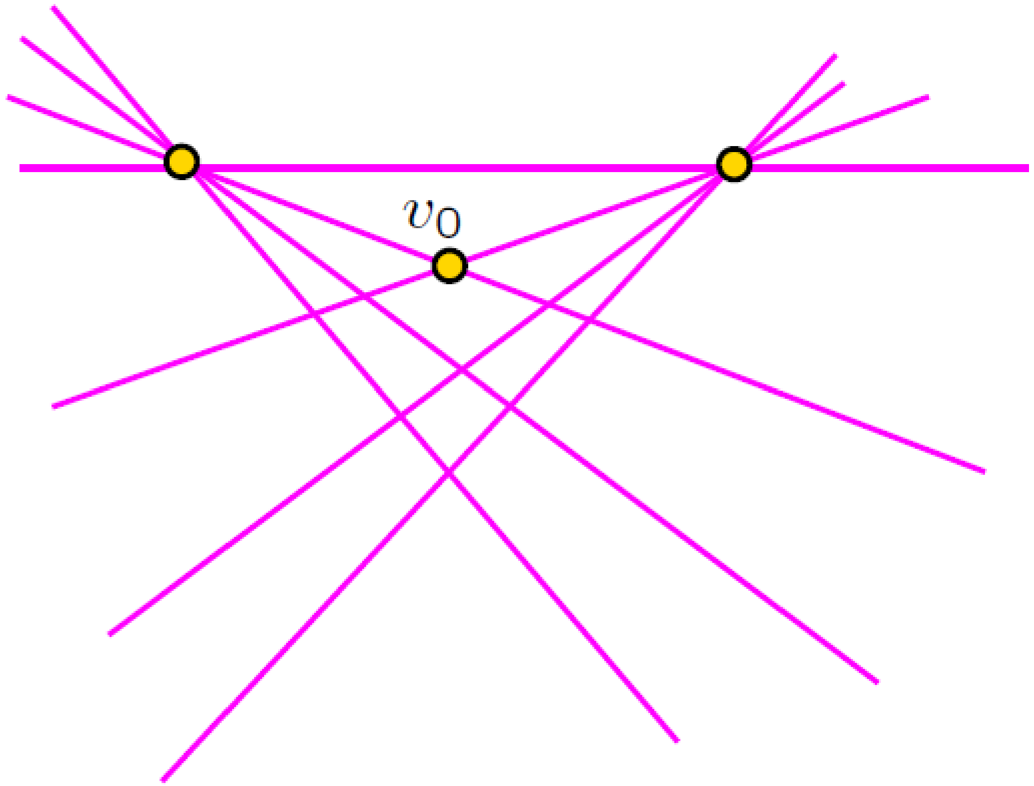
[CGAA]

- The level of a vertex in the arrangement (here): the number of lines strictly below it

Under the general position assumption



The degenerate case



Dan Halperin, Sariel Har-Peled, Kurt Mehlhorn, Eunjin Oh, Micha Sharir:
The Maximum-Level Vertex in an Arrangement of Lines. CoRR abs/2003.00518 (2020)

Arrangements of non-linear objects

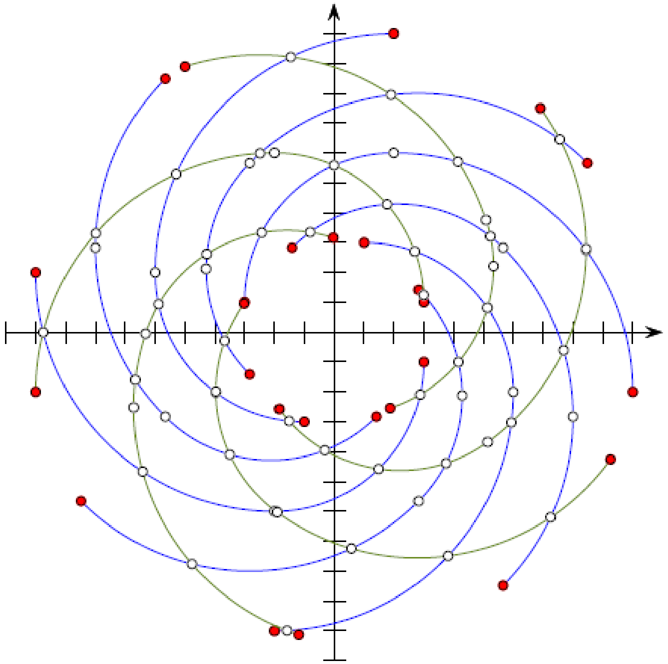


Fig. 5.6: An arrangement induced by a parastichy pair of Fibonacci spirals. The clockwise-winding set (blue) and counterclockwise-winding set (green) consists of eight and five spirals, respectively. Each spiral arc in both sets comprises two circular arcs, the fifth and the sixth circular arcs in the underlying complete Fibonacci spiral.

[FHW]

THE END