# Course Overview

Computational Geometry, Spring 2022

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# Slides overview

- Central predicate: the orientation test
- Course mechanics
- Team
- Bird’s eye view of selected topics
- Convex hull in 3D

# Credits

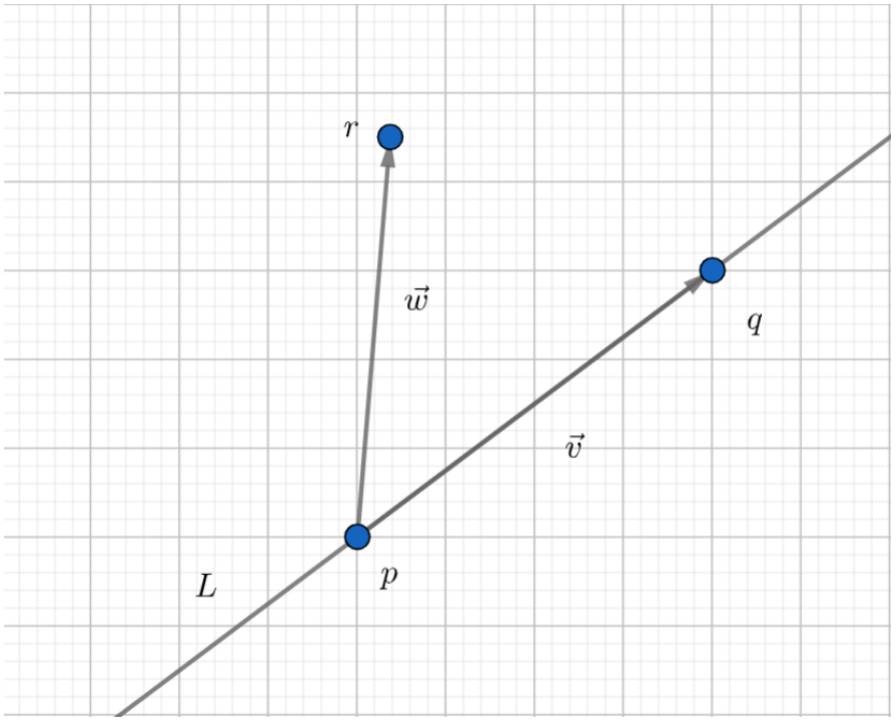
- some figures are taken from Computational Geometry Algorithms and Applications by de Berg et al [CGAA]
- the original figures are available at the book's site: www.cs.uu.nl/geobook/

# The orientation test

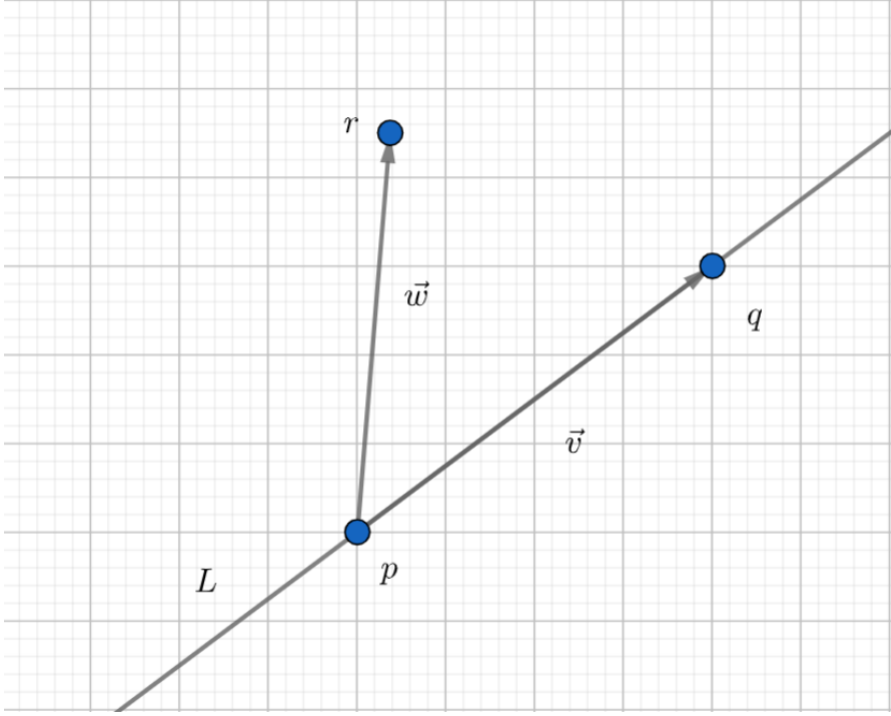
A central predicate, the planar case

# Orientation test

- given three points in the plane $p, q, r$, consider the line $L$ through $p$ and $q$ oriented from $p$ to $q$
- orientation (or side-of-line) test: is $r$ to the left of $L$, on $L$, or to the right of $L$?



# Orientation test, cont'd



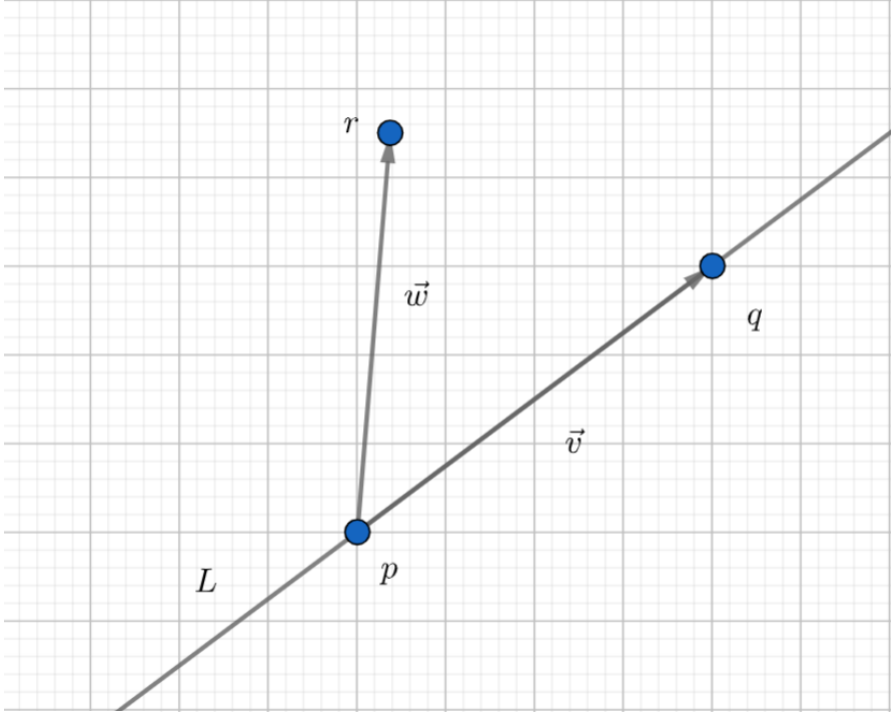

the vector product of $\vec{v}$ and $\vec{w}$:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ w_x & w_y & 0 \end{vmatrix} = (v_x w_y - v_y w_x)\hat{k}$$

$$\vec{v} = q - p \quad \Rightarrow \quad v_x = q_x - p_x, \quad v_y = q_y - p_y$$
$$\vec{w} = r - p \quad \Rightarrow \quad w_x = r_x - p_x, \quad w_y = r_y - p_y$$

$$(v_x w_y - v_y w_x) = (q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x) \equiv \Delta(p, q, r)$$

# Orientation test, cont'd




if $\Delta(p, q, r) > 0$ then $r$ is to the **left** of $L(p, q)$

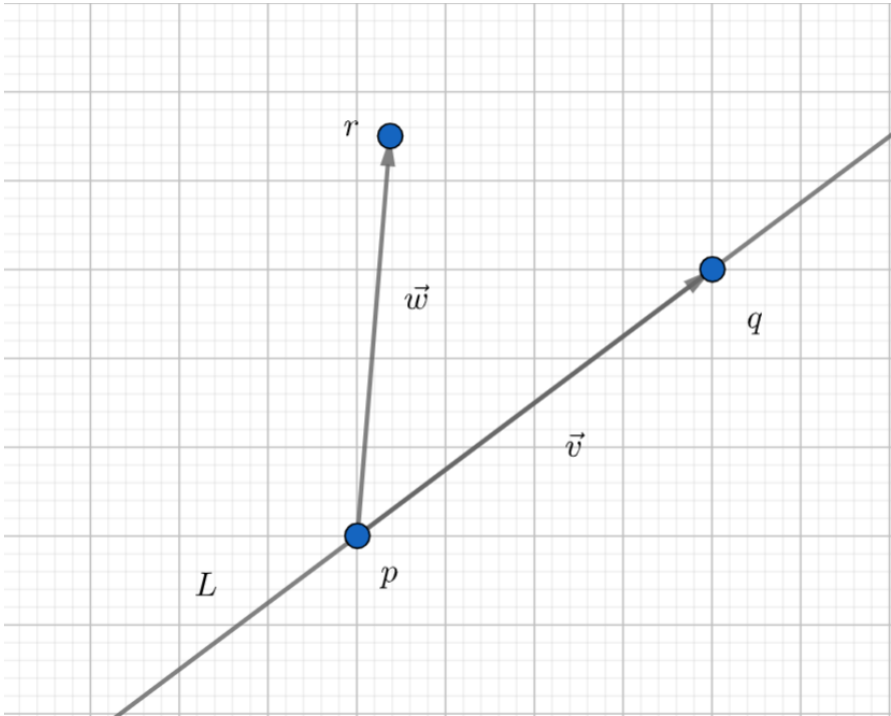
if $\Delta(p, q, r) = 0$ then $r$ is **on** of $L(p, q)$

if $\Delta(p, q, r) < 0$ then $r$ is to the **right** of $L(p, q)$

GeoGebra

# Orientation test, equivalent formulation

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ w_x & w_y & 0 \end{vmatrix} = \begin{vmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{vmatrix}$$



# Orientation test in higher dimensions

- in 3D: on which side of the *oriented plane* $H(p, q, r)$ does the point $s$ lie?

$$\begin{vmatrix} p_x & p_y & p_z & 1 \\ q_x & q_y & q_z & 1 \\ r_x & r_y & r_z & 1 \\ s_x & s_y & s_z & 1 \end{vmatrix} \quad >, \quad <, \quad = \quad 0?$$

- in $R^d$: on which side of an oriented hyperplane containing $d$ points does another point lie? the determinant of a $d + 1 \times d + 1$ matrix

# Course mechanics

# Assignments, theory

- Mandatory! You must submit all the assignments and get a passing grade in each set in order to take the exam
- Five (or four) assignment sets throughout the semester
- Submission via Moodle
- Typed submissions preferred
- It is OK to discuss the assignments with others
- You must write down yourself the solution to each assignment
- The assignment grade is 10% of the final grade and only if it improves the final grade (מגן)

# Programming project, optional

- Will be announced soon
- Large scale
- Can be worked out in pairs
- The project grade is 15% of the final grade and only if it improves the final grade (מגן)

# Final grade composition

- 90% final exam
- 10% assignments

or

- 75% final exam
- 15% programming project
- 10% assignments

# Course website

http://acg.cs.tau.ac.il/courses/computational-geometry/spring-2022/CG-Spring-22

- assignments
- bibliography
- brief lesson summary
- additional information

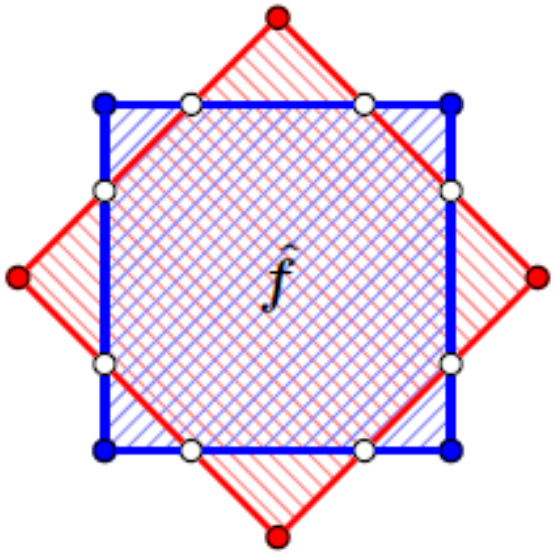
# Course team

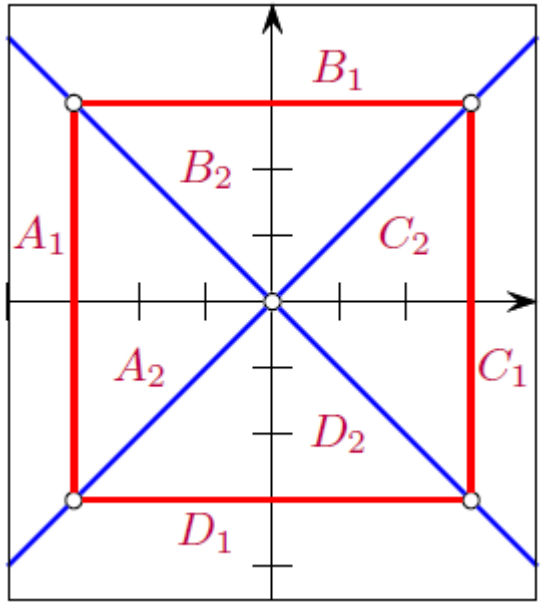
- Instructor: Dan Halperin
- TA: Michal Kleinbort
- Grader: Tal Levi

# Selected topics

Bird’s eye view

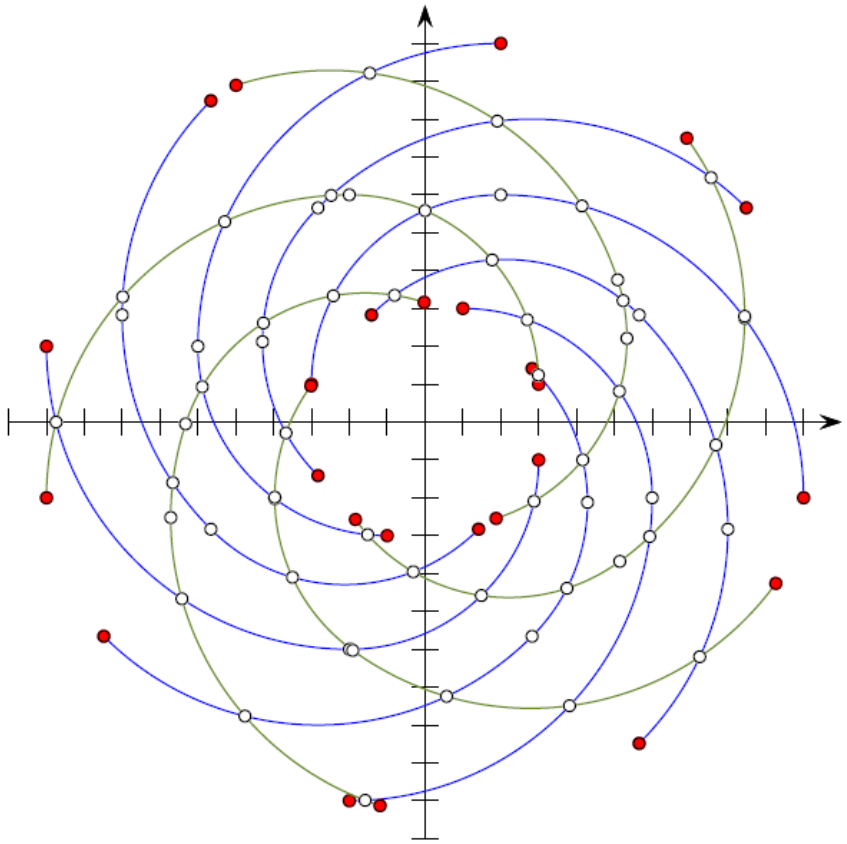
# Map overlay




[CGAL arrgs and their applications, FHW]

# Map overlay, cont’d



[CGAL arrgs and their applications, FHW]

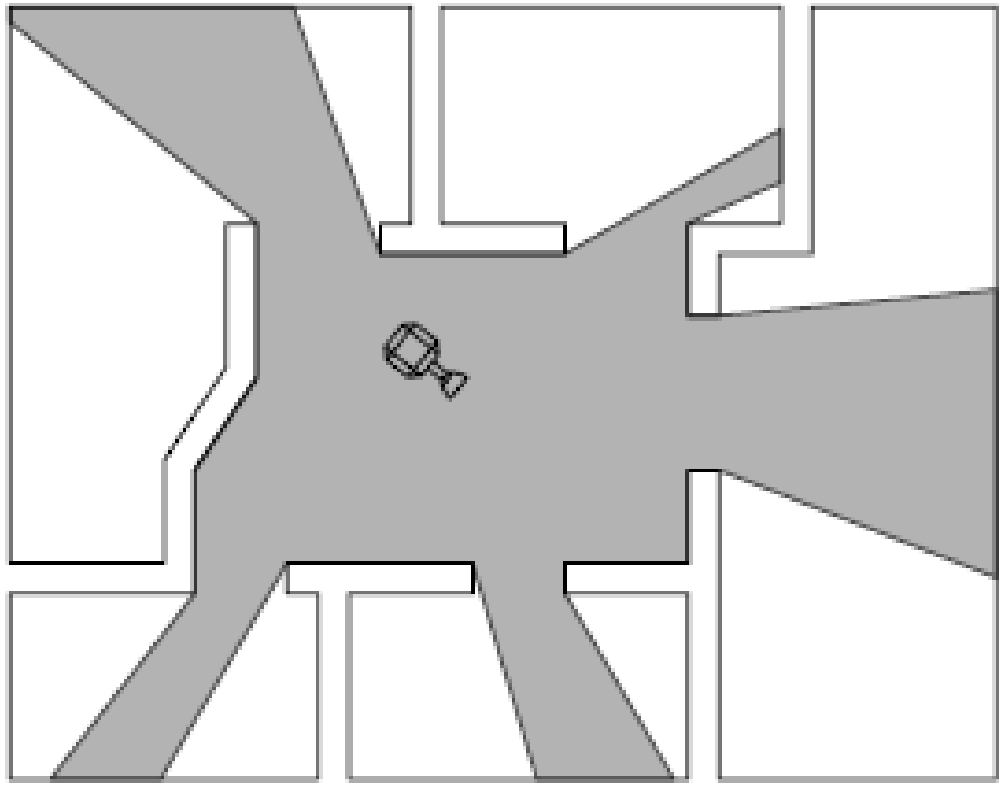
# Map overlay, more example

- potential agricultural pollution
- design plan vs. drone maps
- compare Brazil and Australia

- Also, exposes both a representation of arbitrary two-dimensional entities (generalization of polygons) and a fundamental efficient algorithmic padarigm

# Art gallery and polygon triangulation

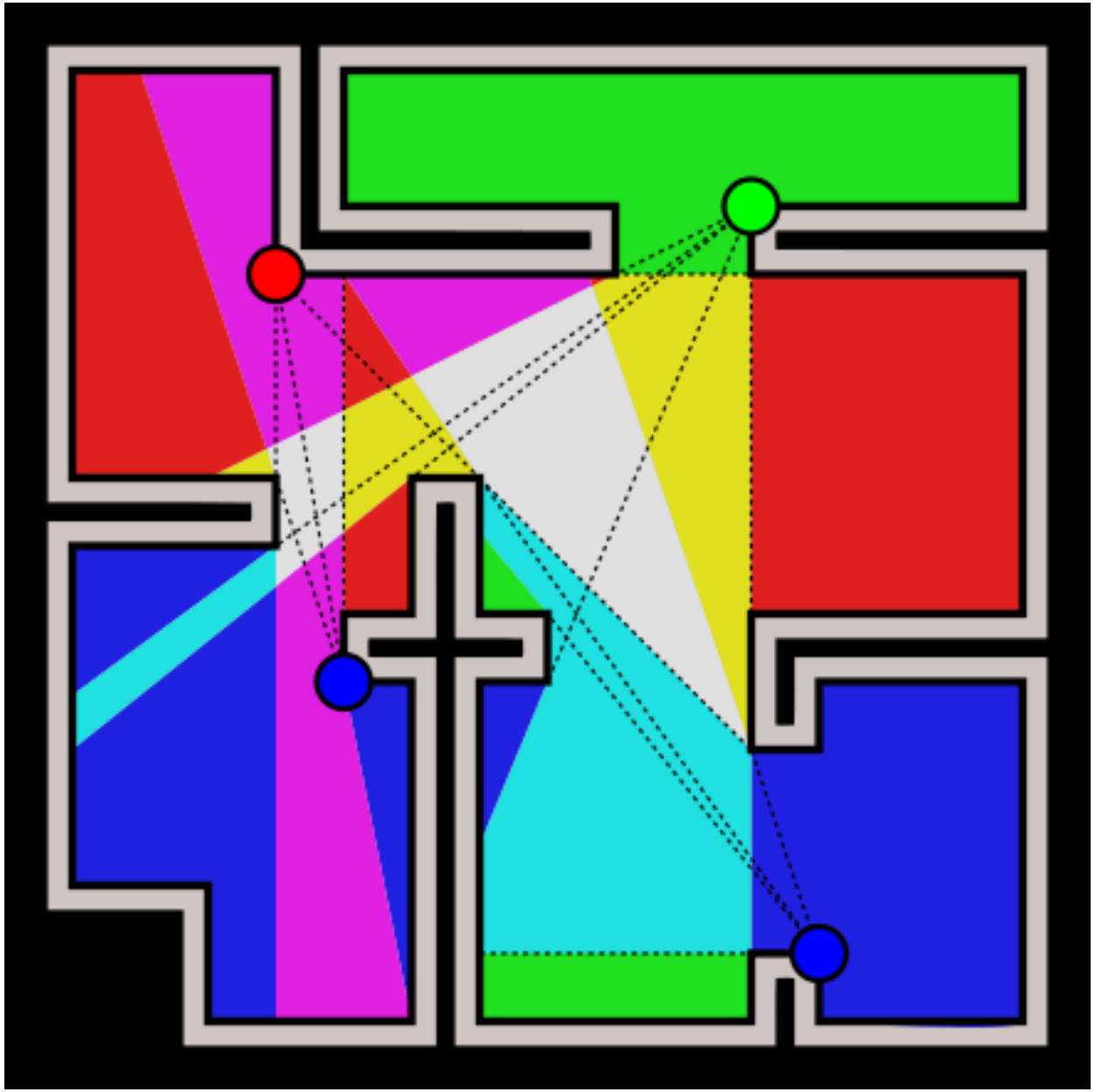
- How many cameras are needed to cover the art gallery?



[CGAA]

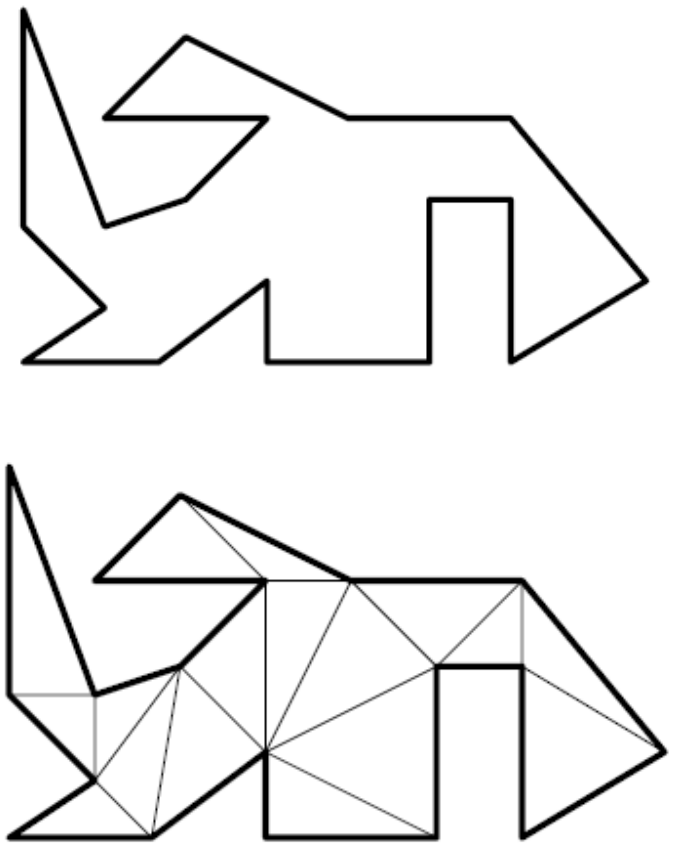
# Art gallery, cont’d

- Four cameras cover this art gallery

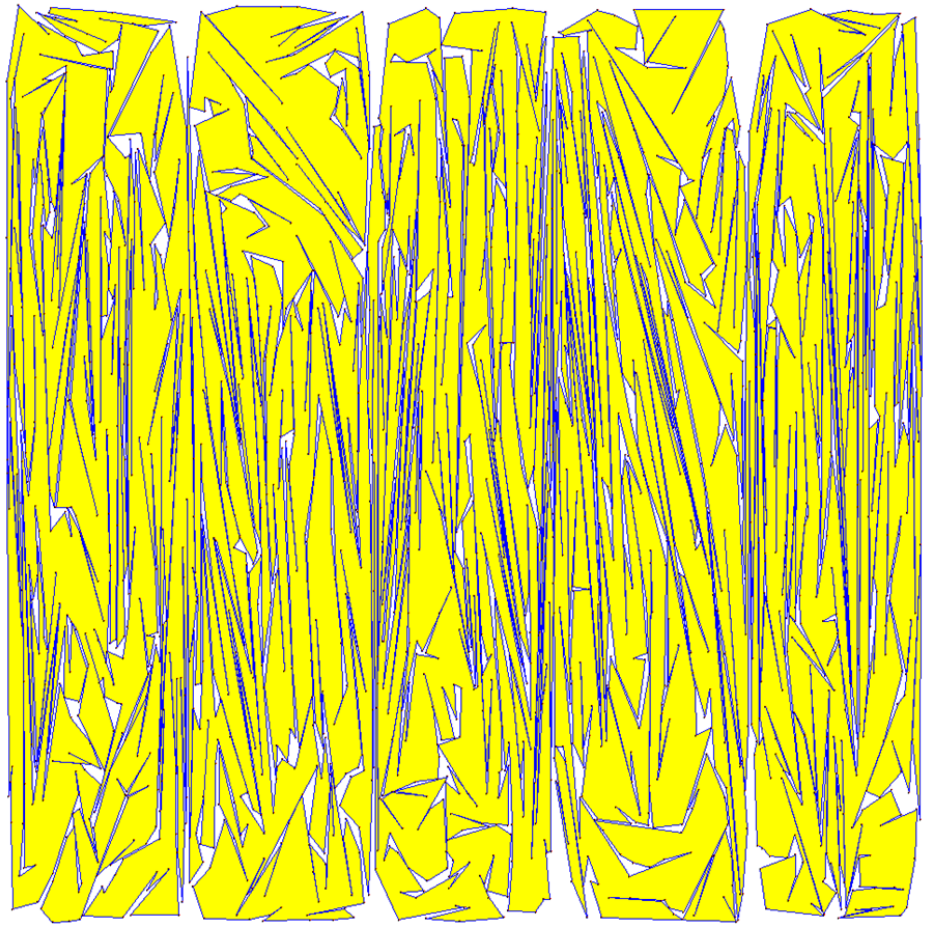


[Wikipedia:art gallery problem]

# Art gallery and polygon triangulation, cont'd

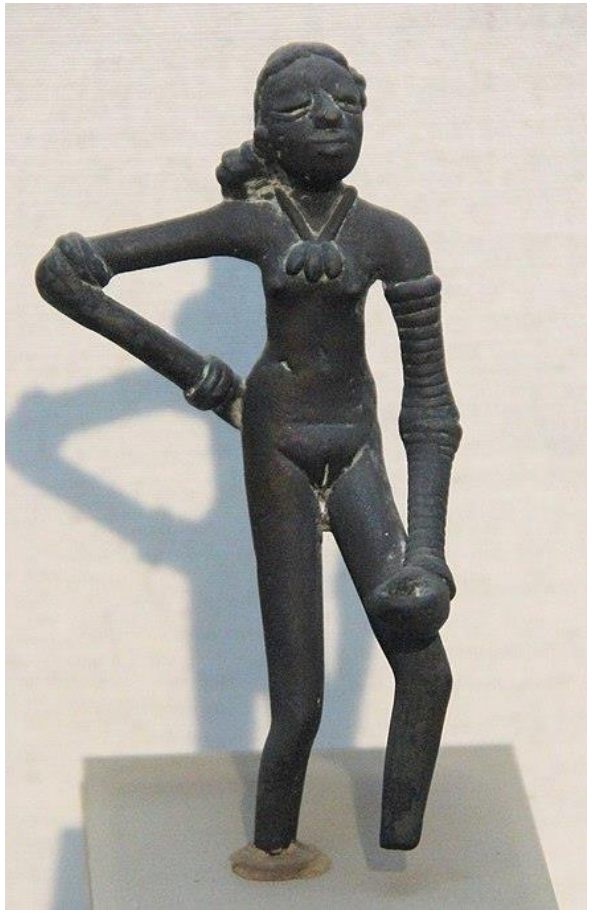


[CGAA]

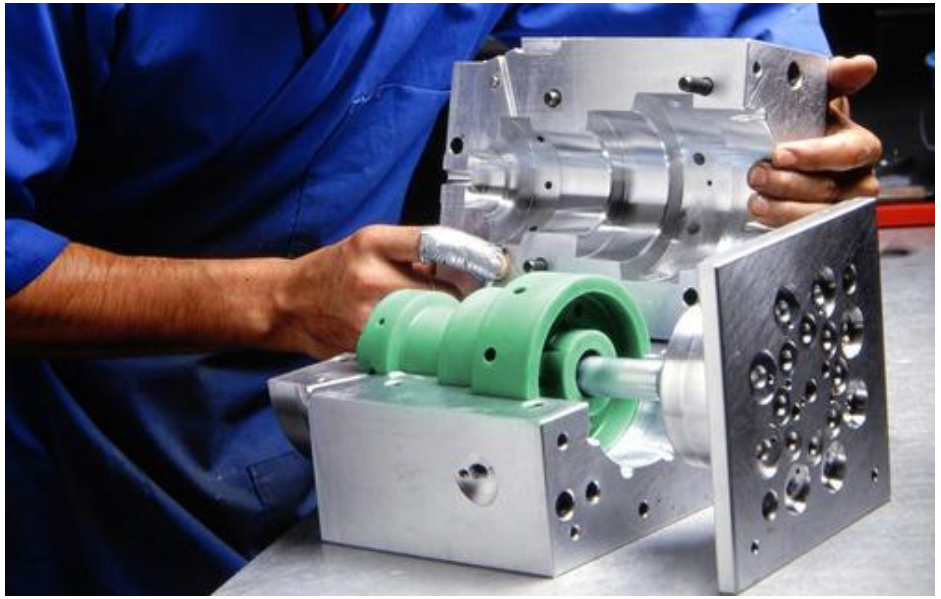


[CG optimization competition, GFH]

# Casting and linear programming



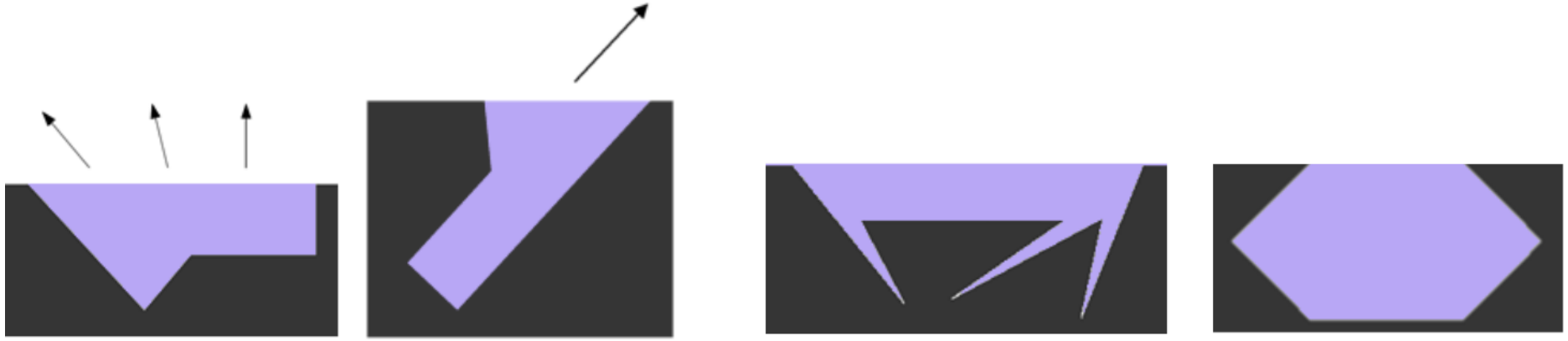
[wikipedia:casting]



[intercast.com:how it’s made]

# Casting and linear programming, cont’d

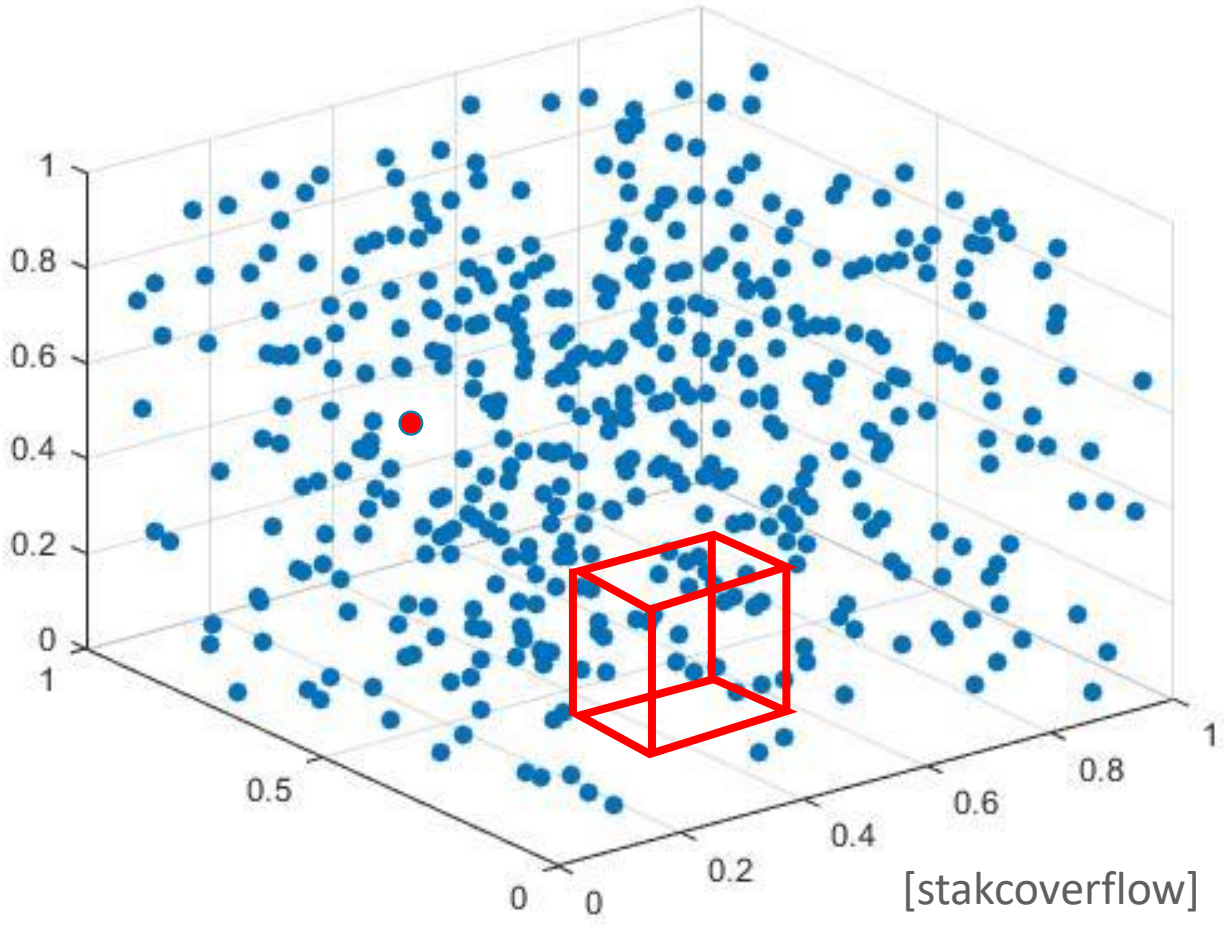
- Can a cast object (polyhedron) be taken out of its mold without breaking the mold?



- Intersection of half-spaces
- Linear programming

# Orthogonal range search and nearest-neighbor search

- Nearest-neighbor search
- Orthogonal range search




[stakcoverflow]

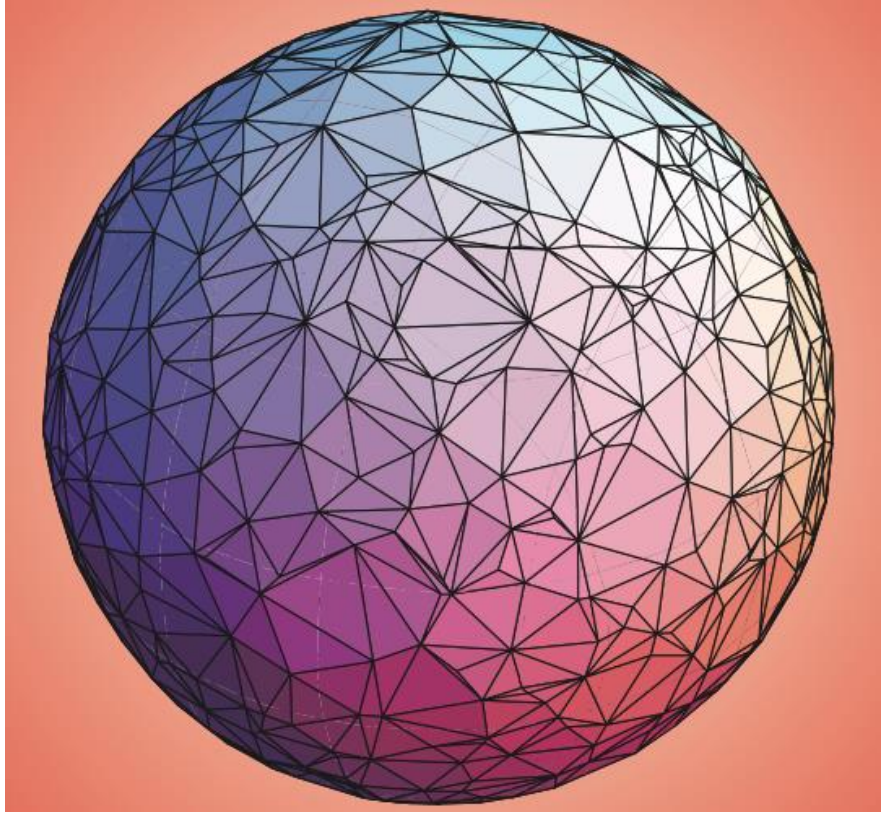
# and more …

- Voronoi diagrams
- Delaunay triangulations
- Smallest enclosing disc
- Point location

# Convex hull in 3D

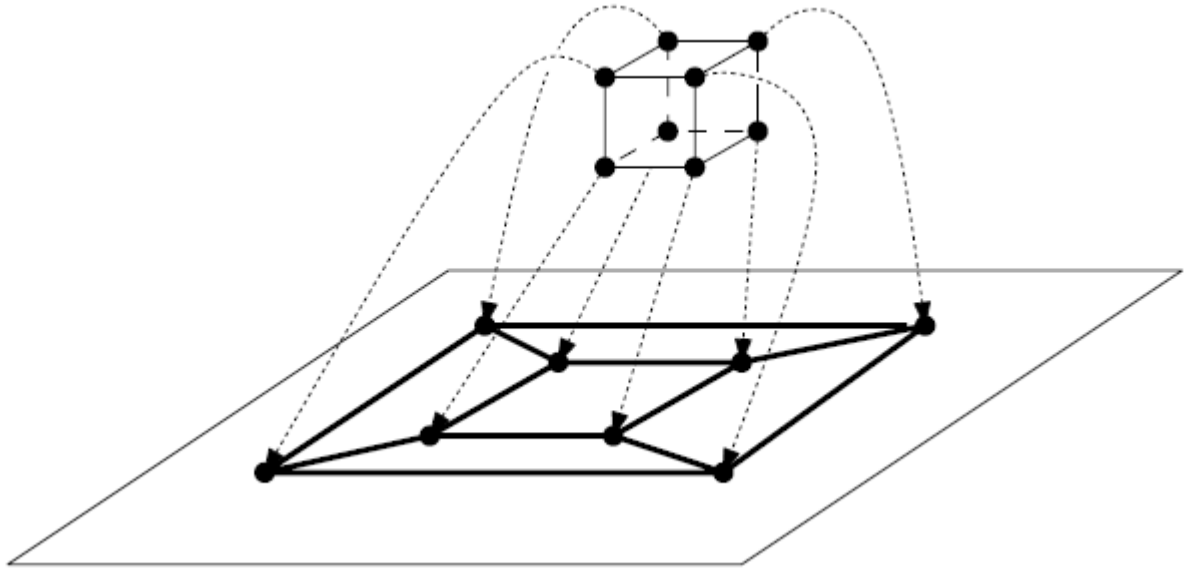
# Convex hull in 3D

- the convex hull of a set $P$ of $n$ points in $R^3$ is a convex polytope whose vertices are points in $P$
- it therefore has at most n vertices
- its vertices and edges constitute a planar graph
- $CH(P)$ has at most $2n - 4$ faces and at most $3n - 6$ edges



[O'Rourke]

# Convex polytopes and planar graphs



- the complexity bounds hold also for non-convex polytopes of *genus* zero with $n$ vertices

# THE END

[Jeb Gaither, CGAL arrangements]