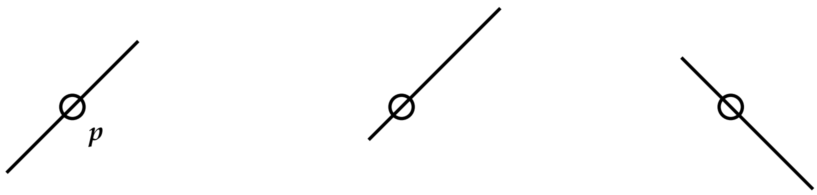
Assignment no. 2

due: Monday, February 12th, 2024

Part I

Exercise 2.1 A Roomba is moving in a room with convex polygonal obstacles, which are pairwise interior-disjoint, with a total of n vertices.
(a) Show that the set of C-obstacles (namely configurations-space obstacles, or expanded obstacles) each corresponding to one convex polygonal obstacle, is a set of pseudo-discs.
(b) optional (bonus) Prove that the complexity of the free space in this case is $O(n)$. Do not rely on the general result for pseudo-discs; however, you can use the results that we have seen for the special cases of discs and/or of polygons.

Exercise 2.2 **(a)** What is the maximum combinatorial complexity (give asymptotic lower and upper bounds) of the free space of the following planar motion-planning problem. A robot arm with two degrees of freedom moving in the plane among polygonal obstacles with a total of n vertices. The arm consists of a line segment that passes through a point p in the plane. It can rotate around p and translate through p, but at all times it coincides with p. See the following figure for an illustration.

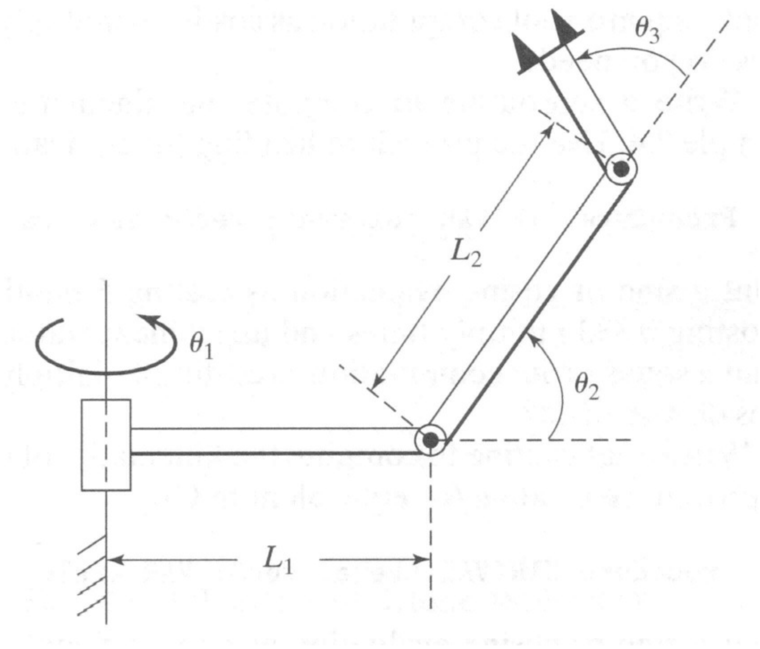


(b) Devise an algorithm to compute the free space. Describe the algorithm in detail. No need to give details on algebraic operations, such as the intersection of two curves.

Part II

Solve **one** of the following exercises (2.3 or 2.4). For either exercise you can work and submit in pairs.

Exercise 2.3 (2) The goal of this exercise is to get acquainted with the basics of robot kinematics. The necessary material is covered, for example, in Chapters 3 (forward kinematics) and 4 (inverse kinematics) of Craig's book, *Introduction to Robotics: Mechanics and Control*. The exercise relates to the following robot arm (see next page) with three rotational degrees of freedom (3R for short; the figure is taken from Craig's book):



(a) (2) Assign coordinate frames to the links of the 3R arm and extract the Denavit-Hartenberg parameters of it. Then write the direct kinematics equations for the arm.

(b) (2) Solve the inverse kinematics problem for the 3R arm as above.

Exercise 2.4 (p2) We are given a simple polygon A, the robot, which can translate in the plane, and a set of pairwise interior-disjoint simple polygons, the obstacles, which the robot has to avoid. We are also given a pair of planar points s and g denoting the reference point of A at a start position and a desired goal position respectively.

(a) (p2) Write a program that solves this motion-planning problem. The program decides whether a *semi-free path*[1] for A from start to goal exists and if so outputs such a path. In this exercise we experiment with exact solutions to motion planning.

In the course's webpage you will find helpful information for solving this problem, as well as the input/output format. In particular you will need to install the full DiscoPygal package.

(b) (p2), optional (bonus) Same as Exercise 2.4(a), only this time you are required to produce a *good* path. Choose your criterion: length, clearance, smoothness, etc. and devise a path that is good according to this criterion; not necessarily optimal, but better than an arbitrary path that the method that puts a point inside each trapezoid and on each vertical edge produces.

[1]Semi-free path means that the robot is allowed to touch the obstacles, but not to penetrate into them.