Assignment no. 3

due date: Monday, February 26th, 2024

All the exercises in this assignment refer to the setting of a line segment (rod, ladder) translating and rotating among (not necessarily convex) polygonal obstacles in the plane.

Exercise 3.1 (p2), 45 points The **rod** component of the DiscoPygal software package, which is available for download from the course website (you need to download the new version), comes with a rudimentary PRM solver, `basicRodPRM`, for the rod-among-polygons motion planning problem.
Possible improvements to the solver include, but are not limited to: careful choice of the number of neareast-neighbors to connect to (or if you use neighbors in a ball—the radius of the ball), more efficient collision detection, lowering the sampling rate while still being conservative.[1]
(a) Choose one aspect of improvement (or more if you wish), and implement it. Write a brief description of your PRM planner and its major improvements over the original given version.
(b) There are a couple of scenarios in the package that you download. Design a few more (three or more) scenarios of varying difficulty for your planner, from easy to hard, including at least one instance that is densely populated with obstacles.
(c) Design experiments to compare between the performance of the original solver and your improved version. Report on the experimental results.
(d) Try two different distance measures (of your choice/design) between a pair of configurations. Design, carry out, and report on experiments that show the effect of the choice of a distance measure on the performance of the planner.

Exercise 3.2 (p2), 30 points Implement an RRT solver for the rod problem. (You can use ingredients from your PRM solver.) Briefly describe your RRT planner. Compare the performance of your PRM vs. your RRT solver, and report on experimental results.

Exercise 3.3, 25 points Assume the rod is translating and rotating among polygons with a total of n vertices. In class we saw a representation of the free space for the rod-among-polygons problem, the complexity of which is $O(n^5)$. The actual complexity of the free space in this motion-planning instance is much lower.
(a) Show that the maximum combinatorial complexity of the free configuration space in this case is $O(n^2)$. To show this bound you have to bound the number of semi-free triple contacts (namely placements of the rod where it touches the obstacles boundaries in three points without penetrating into the obstacles). From February 16th, you can get a hint (for free) by sending me an email (`danha@tauex.tau.ac.il`) with the subject line "hint for Ex 3.3"
(b) Show that the above bound is tight in the worst case. That is, describe a scene where the complexity of the free space is $\Omega(n^2)$.

[1] A basic procedure in sampling-based motion planning is the so-called *local planner*, which decides whether a straight line segment in configuration space is collision free, typically by sampling a finite number of points along the segment and testing each sample for collision. In order to be conservative in local planning, we expand the robot or the obstacles by a small-radius ball that will guarantee that if all samples along a straight line segment are collision-free then the robot is guaranteed not to collide with the obstacles also in-between samples. Once we determine the size of a small disc to expand the polygonal obstacles with, we wish to sample as few samples as possible that will still guarantee conservativeness.