Computational Geometry

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Computational Geometry Algorithm Library
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Outline

1. CGAL
   - Introduction
   - Content
   - Literature
   - Geometry Factory
   - Details

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Outline

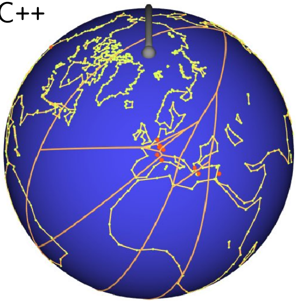
CGAL: Mission

"Make the large body of geometric algorithms developed in the field of computational geometry available for industrial applications"

CGAL Project Proposal, 1996

CGAL Facts

- A collection of software packages written in C++
- Adheres the *generic programming* paradigm
- Development started in 1995
- An open source library
- Several active contributor sites
- High search-engine ranking for www.cgal.org



- Used in a diverse range of domains
  - e.g., computer graphics, scientific visualization, computer aided design and modeling, additive manufacturing, geographic information systems, molecular biology, medical imaging, and VLSI
- The de-facto standard in applied Computational Geometry

CGAL in Numbers

600,000	lines of C++ code
10,000	downloads per year not including Linux distributions
4,500	manual pages (user and reference manual)
1,000	subscribers to user mailing list
300	commercial users
150	packages
30	active developers
6	months release cycle
2	licenses: Open Source and commercial

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CGAL History

Year	Version Released	Other Milestones
1996		CGAL founded
1998	July 1.1	
1999		Work continued after end of European support
2001	Aug 2.3	Editorial Board established
2002	May 2.4	
2003	Nov 3.0	GEOMETRY FACTORY founded
⋮		
2008		CMAKE
2009	Jan 3.4, Oct 3.5	
2010	Mar 3.6, Oct 3.7	Google Summer of Code (GSoC) 2010
2011	Apr 3.8, Aug 3.9	GSoC 2011
2012	Mar 4.0, Oct 4.1	GSoC 2012
2013	Mar 4.2, Oct 4.3	GSoC 2013, Doxygen
2014	Apr 4.4, Oct 4.5	GSoC 2014
2015	Apr 4.6, Oct 4.7	GitHub, HTML5, Main repository made public
2016	Apr 4.8, Sep 4.9	Only headers, 20th anniversary
2017	May 4.10, Sep 4.11	CTEST, GSoC 2017
2018	Apr 4.12, Oct 4.13	GSoC 2018; Basic viewers
2019	Nov 5.0	C++14, GSoC 2019
2020	Sep 5.1, Dec 5.2	GSoC 2020
2021	Jun 5.3	GSoC 2021
2022	Jan 5.4	GSoC 2022

CGAL Properties

- Reliability
  - Explicitly handles degeneracies
  - Follows the Exact Geometric Computation (EGC) paradigm
- Efficiency
  - Depends on leading 3rd party libraries
    - ⋆ e.g., BOOST, GMP, MPFR, QT, EIGEN, TBB, and CORE
  - Adheres to the generic-programming paradigm
    - ⋆ Polymorphism is resolved at compile time

The best of both worlds

CGAL Properties, Cont

- Flexibility
  - Adaptable, e.g., graph algorithms can directly be applied to CGAL data structures
  - Extensible, e.g., data structures can be extended
- Ease of Use
  - Has didactic and exhaustive Manuals
  - Follows standard concepts (e.g., C++ and STL)
  - Has a modular structure, e.g., geometry and topology are separated
  - Characterizes with a smooth learning-curve

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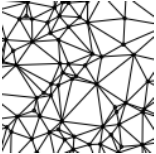
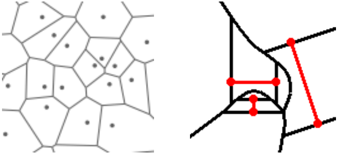
2D Algorithms and Data Structures



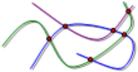
Triangulations

Mesh Generation

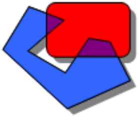
Polyline Simplification



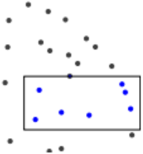
Voronoi Diagrams



Arrangements

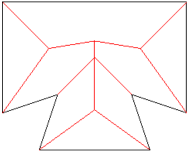


Boolean Operations



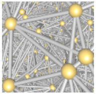
Neighborhood Queries

Minkowski Sums

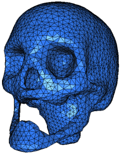


Straight Skeleton

3D Algorithms and Data Structures

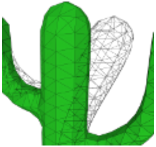


Triangulations

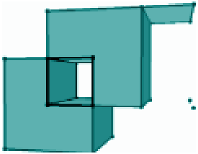


Mesh Generation

Polyhedral Surface

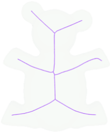


Deformation



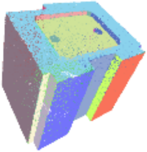
Boolean Operations

Mesh Simplification

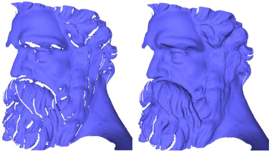


Skeleton

Segmentation



Classification



Hole Filling

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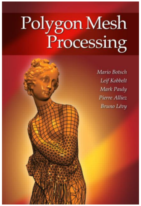
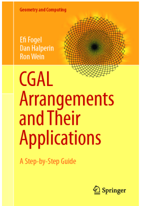
CGAL Bibliography I

- The CGAL Project.
  CGAL User and Reference Manual.
  CGAL Editorial Board, 4.4 edition, 2014. http://doc.cgal.org/4.2/CGAL.CGAL/html/index.html
- Efi Fogel, Ron Wein, and Dan Halperin.
  CGAL Arrangements and Their Applications, A Step-by-Step Guide.
  Springer, 2012.
- Mario Botsch, Leif Kobbelt, Mark Pauly, Pierre Alliez, and Bruno Levy.
  Polygon Mesh Processing.
  CRC Press, 2010.
- A. Fabri, G.-J. Giezeman, L. Kettner, S. Schirra, and S. Schönherr.
  On the design of CGAL a computational geometry algorithms library.
  Software — Practice and Experience, 30(11):1167–1202, 2000. Special Issue on Discrete Algorithm Engineering.
- A. Fabri and S. Pion.
  A generic lazy evaluation scheme for exact geometric computations.
  In 2*nd* *Library-Centric Software Design Workshop*, 2006.
- M. H. Overmars.
  Designing the computational geometry algorithms library CGAL.
  In *Proceedings of ACM Workshop on Applied Computational Geometry, Towards Geometric Engineering*, volume 1148, pages 53–58, London, UK, 1996. Springer.
- Many Many Many papers





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Some CGAL Commercial Customers

Agilent Technologies
Orbotech
pulsic
ECT
SAMSUNG
Ceradrop
CMET シーメット株式会社
Autodesk
powel
ECL
Bentley

3D printing
CAD
VLSI
medical
engineering
GIS
oil,gas, mining
telecom

zimmer
medicim
St. Jude Medical
ANSYS
FEI
The MathWorks
Industrial Research Limited
erdas
Virtualwind
SAFE SOFTWARE
Trimble
ESRI
ENVITIA
PETROBRAS
ExxonMobil
brgm
Schlumberger
TOTAL
BT
orange
أرامكو السعودية Saudi Aramco
Midland Valley
RioTinto

A.Fabri / Total / Oct '16

CGAL Commercial Customers, Geographic Segmentation

Outline

CGAL Structure

Basic Library

Algorithms and Data Structures
e.g., Triangulations, Surfaces, and Arrangements

Kernel

Elementary geometric objects
Elementary geometric computations on them

Visualization
Files
I/O
Number Types
Generators

Support Library

Configurations, Assertions,...

CGAL Kernel Concept

- Geometric objects of constant size.
- Geometric operations on object of constant size.

Primitives 2D, 3D, dD		**Operations**	
		Predicates	**Constructions**
point	●	comparison	intersection
vector	$\rightarrow$	orientation	squared distance
triangle	$\triangle$	containment	...
iso rectangle	▭	...	
circle	○		
...			

CGAL Kernel Affine Geometry

point - origin	$\rightarrow$ vector
point - point	$\rightarrow$ vector
point + vector	$\rightarrow$ point

point + point	$\leftarrow$ Illegal

$$\text{midpoint}(a, b) = a + 1/2 \times (b - a)$$

CGAL Kernel Classification

- Dimension: 2, 3, arbitrary
- Number types:
  - Ring: $+, -, \times$
  - Euclidean ring (adds integer division and gcd) (e.g., CGAL::Gmpz).
  - Field: $+, -, \times, /$ (e.g., CGAL::Gmpq).
  - Exact sign evaluation for expressions with roots (`Field_with_sqr`).
- Coordinate representation
  - Cartesian—requires a *field* number type or *Euclidean ring* if no constructions are performed.
  - Homegeneous—requires *Euclidean ring*.
- Reference counting
- Exact, Filtered

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CGAL Kernels and Number Types

Cartesian representation	Homogeneous representation
point $x = \frac{hx}{hw}$, $y = \frac{hy}{hw}$	point hx, hy, hw

Intersection of two lines

$$\begin{cases} a_1 x + b_1 y + c_1 = 0 \\ a_2 x + b_2 y + c_2 = 0 \end{cases}$$

$$\begin{cases} a_1 hx + b_1 hy + c_1 hw = 0 \\ a_2 hx + b_2 hy + c_2 hw = 0 \end{cases}$$

$$(x, y) = \left(\frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, -\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \right)$$

Field operations

$$(hx, hy, hw) = \left(\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, -\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right)$$

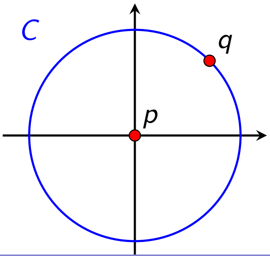
Ring operations

CGAL Numerical Issues

```
#if 1
   typedef CORE::Expr                                    NT;
   typedef CGAL::Cartesian<NT>                           Kernel;
  NT sqrt2 = CGAL::sqrt(NT(2));
#else
   typedef double                                        NT;
   typedef CGAL::Cartesian<NT>                           Kernel;
  NT sqrt2 = sqrt(2);
#endif

 Kernel::Point_2 p(0,0), q(sqrt2,sqrt2);
 Kernel::Circle_2 C(p,4);
 assert(C.has_on_boundary(q));
```

- OK if NT supports exact sqrt.
- Assertion violation otherwise.



CGAL Pre-defined Cartesian Kernels

- Support construction of points from `double` Cartesian coordinates.
- Support exact geometric predicates.
- Handle geometric constructions differently:
  - `CGAL::Exact_predicates_inexact_constructions_kernel`
    - ⋆ Geometric constructions may be inexact due to round-off errors.
    - ⋆ It is however more efficient and sufficient for most CGAL algorithms.
  - `CGAL::Exact_predicates_exact_constructions_kernel`
  - `CGAL::Exact_predicates_exact_constructions_kernel_with_sqrt`
    - ⋆ Its number type supports the exact square-root operation.

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CGAL Special Kernels

- Filtered kernels
- 2D circular kernel
- 3D spherical kernel
- Refer to CGAL's manual for more details.

CGAL Basic Library

- Generic data structures are parameterized with Traits
  - Separates algorithms and data structures from the geometric kernel.
- Generic algorithms are parameterized with iterator ranges
  - Decouples the algorithm from the data structure.

CGAL Components Developed at Tel Aviv University

- 2D Arrangements
- 2D Regularized Boolean Set-Operations
- 2D Minkowski Sums
- 2D Envelopes
- 3D Envelopes
- 2D Snap Rounding
- 2D Set Movable Seperability (2D Casting)
- 3D Set Movable Seperability (3D Casting)
- Inscribed Areas / 2D Largest empty iso rectangle
- CGAL Python bindings for the above

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