Course Overview

Computational Geometry, Fall 2020-2021

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Slides overview

- Central predicate: the orientation test
- Course mechanics
- Team
- CG challenge
- Bird’s eye view of selected topics
- Convex hull in 3D

Credits

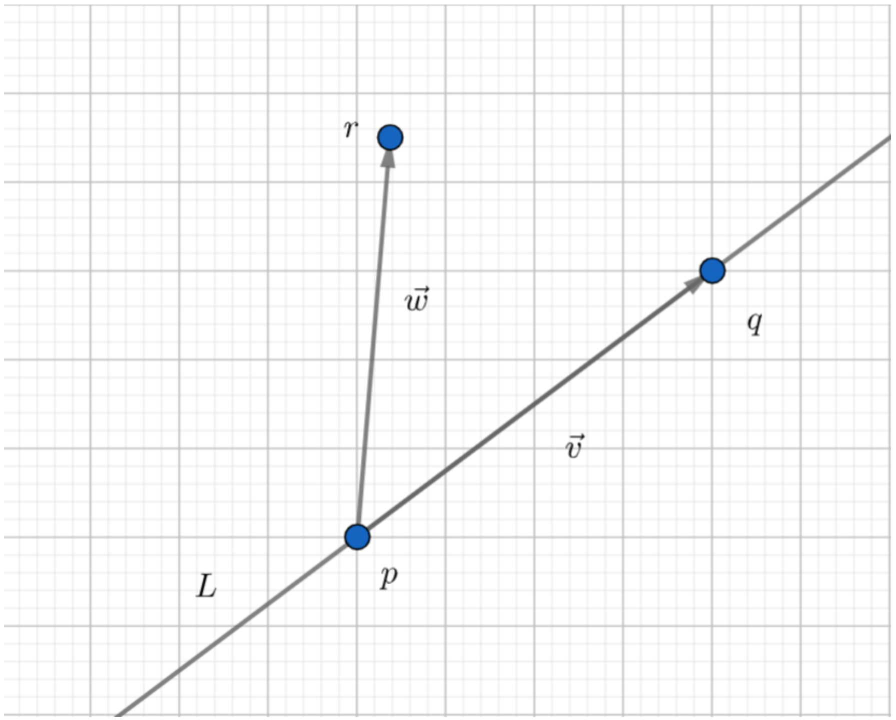
- some figures are taken from Computational Geometry Algorithms and Applications by de Berg et al [CGAA]
- the original figures are available at the book’s site: www.cs.uu.nl/geobook/

The orientation test

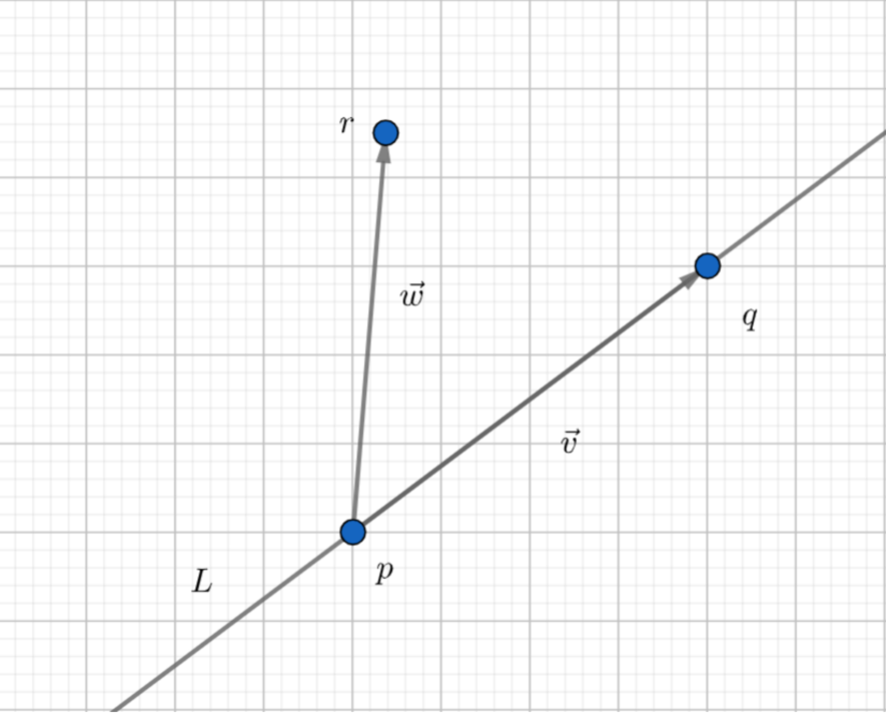
A central predicate, the planar case

Orientation test

- given three points in the plane p, q, r, consider the line L through p and q oriented from p to q
- orientation (or side-of-line) test: is r to the left of L, on L, or to the right of L?



Orientation test, cont'd



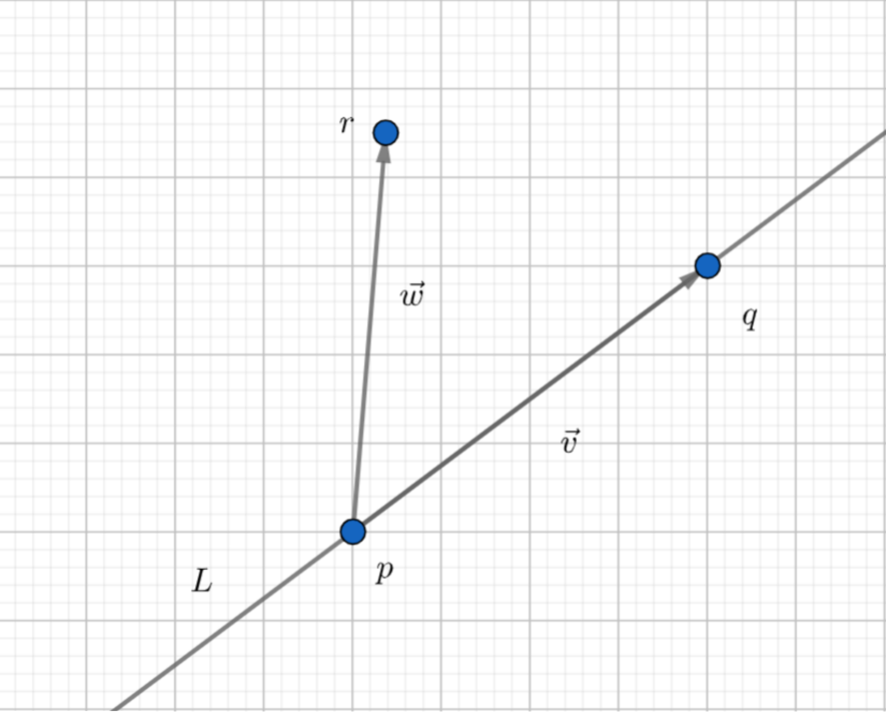
the vector product of $\vec{v}$ and $\vec{w}$:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ w_x & w_y & 0 \end{vmatrix} = (v_x w_y - v_y w_x)\hat{k}$$

$$\vec{v} = q - p \quad \Rightarrow \quad v_x = q_x - p_x, \quad v_y = q_y - p_y$$
$$\vec{w} = r - p \quad \Rightarrow \quad w_x = r_x - p_x, \quad w_y = r_y - p_y$$

$$(v_x w_y - v_y w_x) = (q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x) \equiv \Delta(p, q, r)$$

Orientation test, cont'd



if $\Delta(p, q, r) > 0$ then r is to the **left** of $L(p, q)$

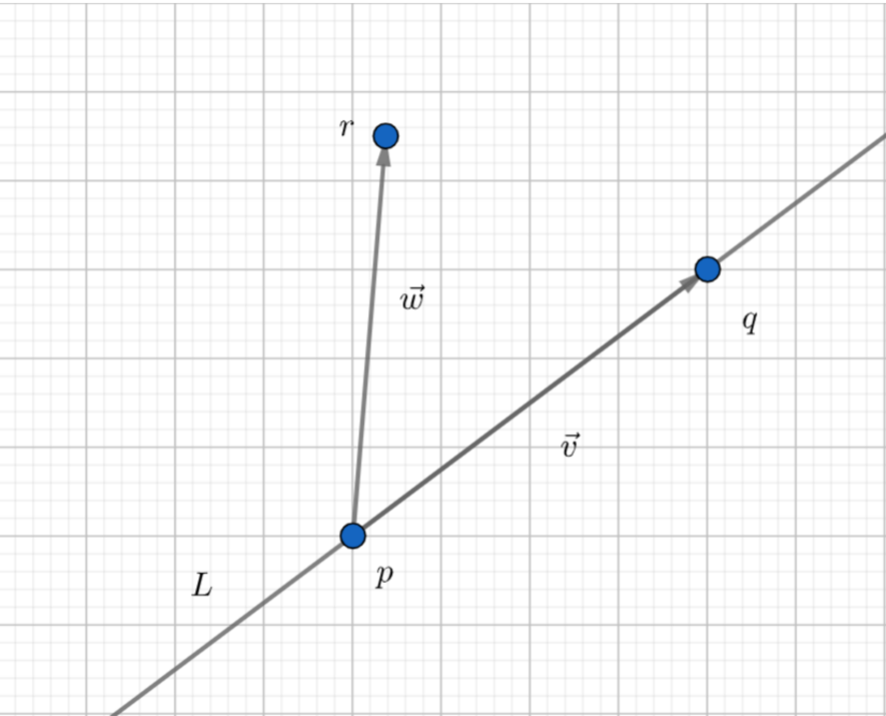
if $\Delta(p, q, r) = 0$ then r is **on** of $L(p, q)$

if $\Delta(p, q, r) < 0$ then r is to the **right** of $L(p, q)$

GeoGebra

Orientation test, equivalent formulation

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ w_x & w_y & 0 \end{vmatrix} = \begin{vmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{vmatrix}$$



Orientation test in higher dimensions

- in 3D: on which side of the *oriented plane* $H(p,q,r)$ does the point s lie?

$$\begin{vmatrix} p_x & p_y & p_z & 1 \\ q_x & q_y & q_z & 1 \\ r_x & r_y & r_z & 1 \\ s_x & s_y & s_z & 1 \end{vmatrix} \quad >, \quad <, \quad = \quad 0?$$

- in R^d: on which side of an oriented hyperplane containing d points does another point lie? the determinant of a $d+1 \times d+1$ matrix

Course mechanics

Assignments, theory

- Mandatory! You must submit all the assignments and get a passing grade in each set in order to take the exam
- Five (or four) assignment sets throughout the semester
- Submission via Moodle
- Typed submissions preferred
- It is OK to discuss the assignments with others
- You must write down yourself the solution to each assignment
- The assignment grade is 10% of the final grade and only if it improves the final grade (מגן)

Programming project, optional

- Will be announced soon
- Large scale
- Can be worked out in pairs
- The project grade is 15% of the final grade and only if it improves the final grade (מגן)

Final grade composition

- 90% final exam
- 10% assignments

or

- 75% final exam
- 15% programming project
- 10% assignments

Course website

http://acg.cs.tau.ac.il/courses/computational-geometry/Fall%202020-2021/CG-Fall-2020-21

- assignments
- bibliography
- brief lesson summary
- additional information

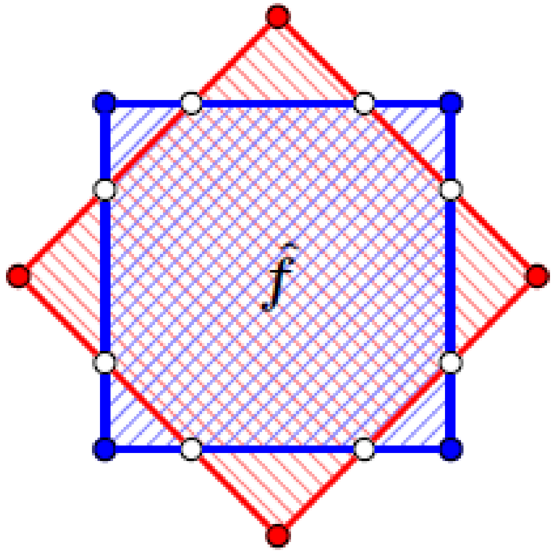
Course team

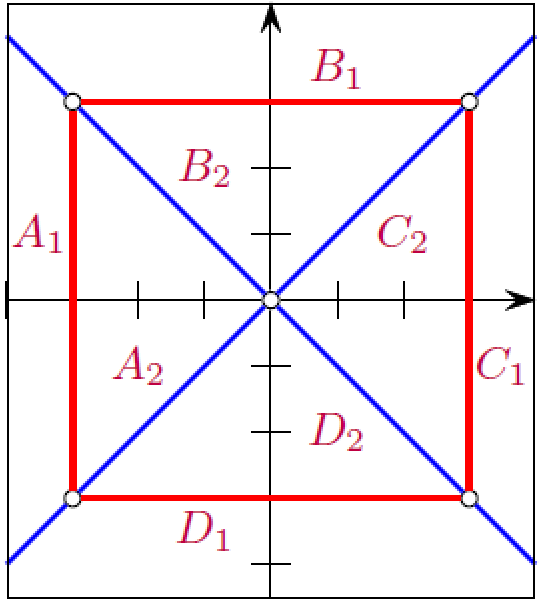
- Instructor: Dan Halperin
- TA: Michal Kleinbort
- Grader: Tomer Even

Selected topics

Bird’s eye view

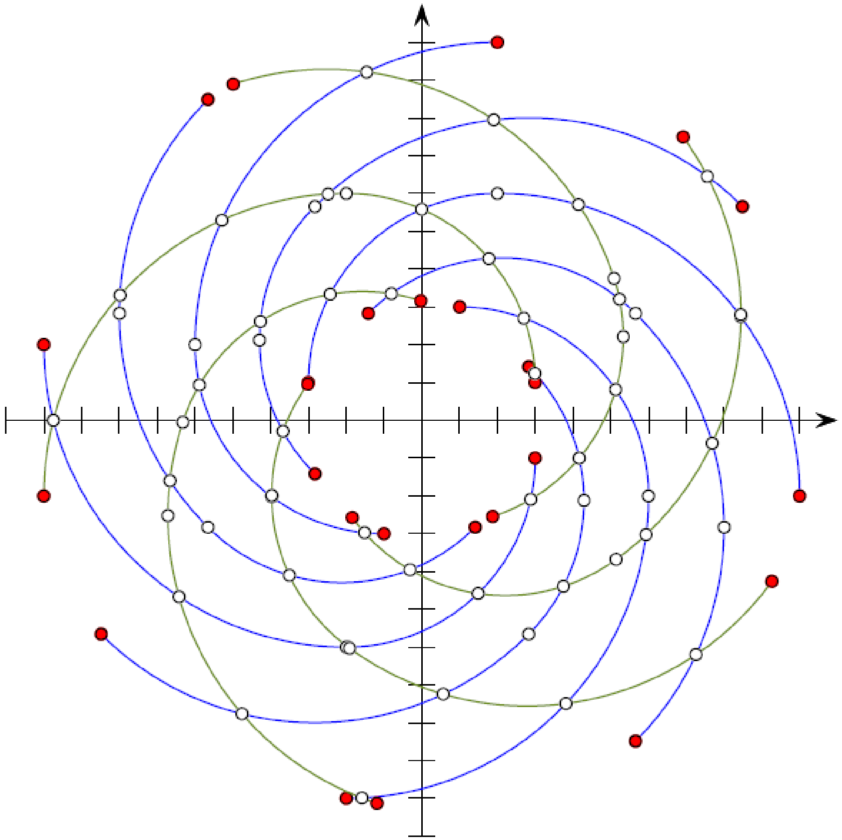
Map overlay





[CGAL arrgs and their applications, FHW]

Map overlay, cont’d



[CGAL arrgs and their applications, FHW]

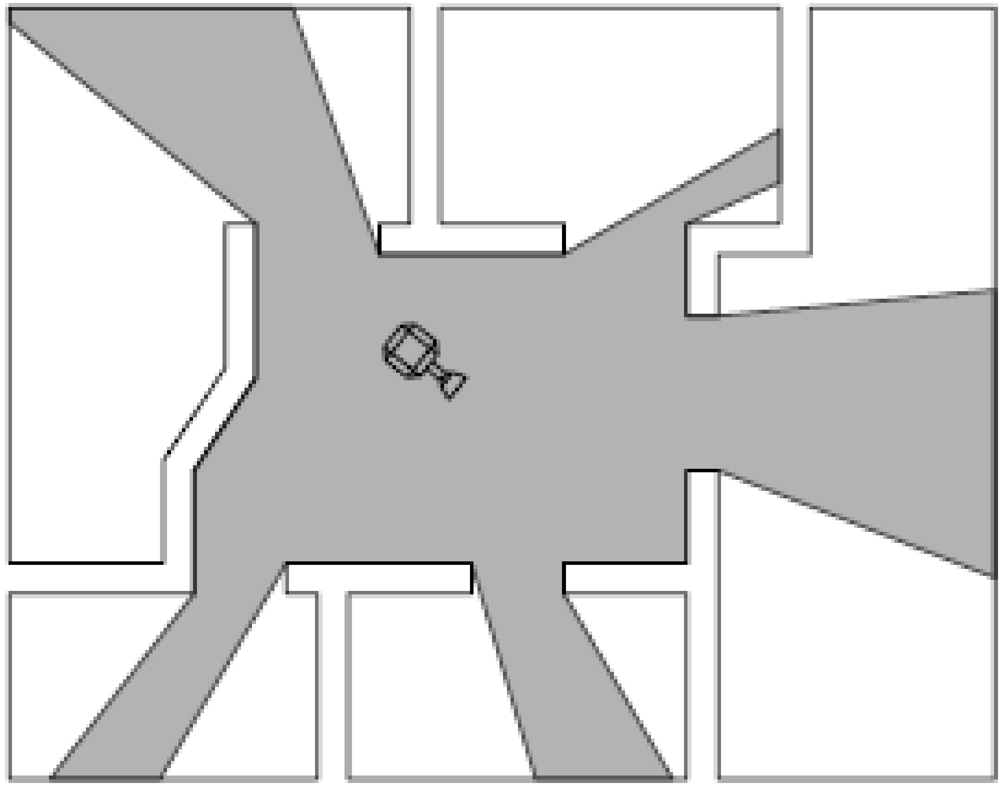
Map overlay, more example

- potential agricultural pollution
- design plan vs. drone maps
- compare Brazil and Australia

- Also, exposes both a representation of arbitrary two-dimensional entities (generalization of polygons) and a fundamental efficient algorithmic padarigm

Art gallery and polygon triangulation

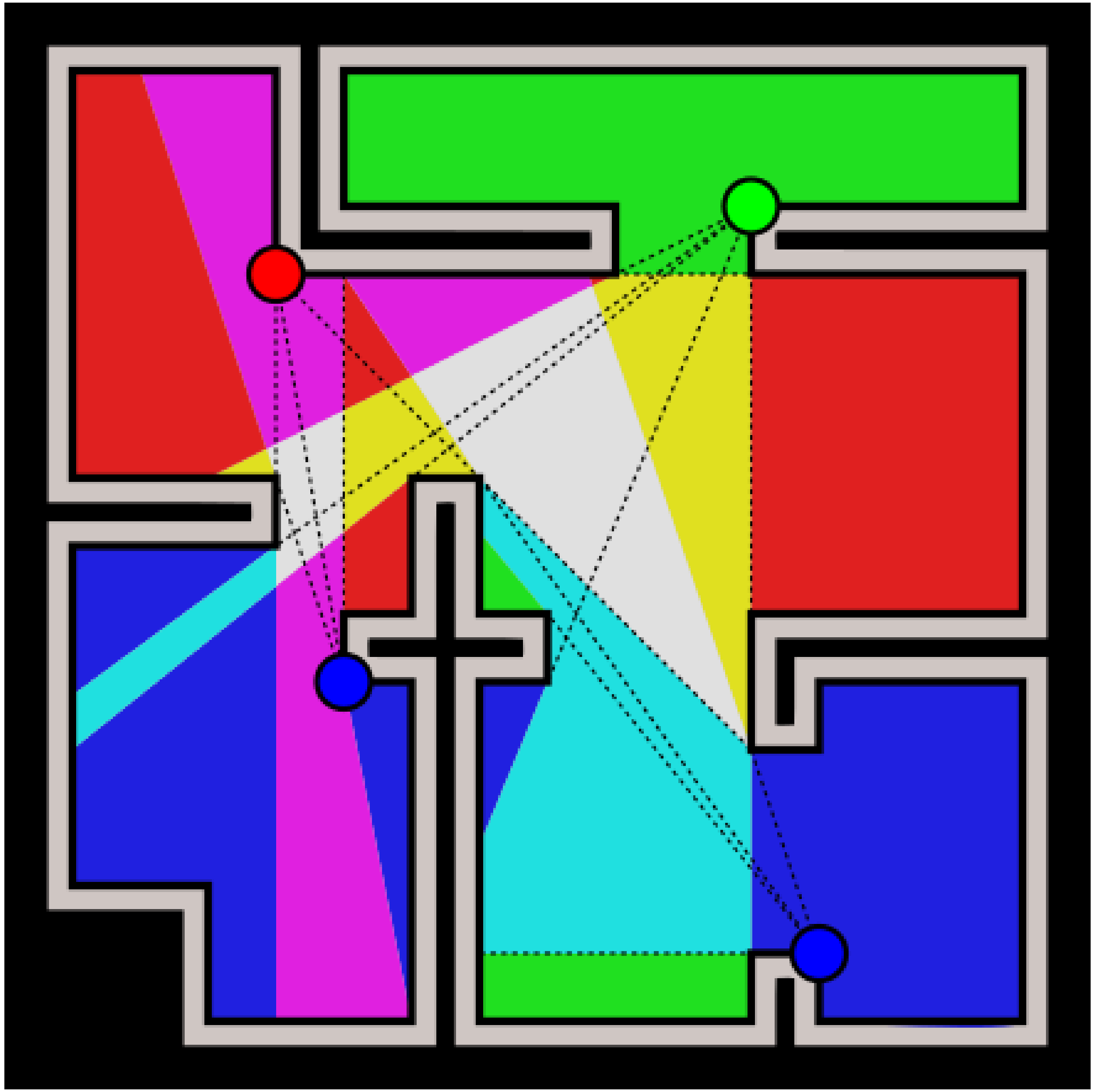
- How many cameras are needed to cover the art gallery?



[CGAA]

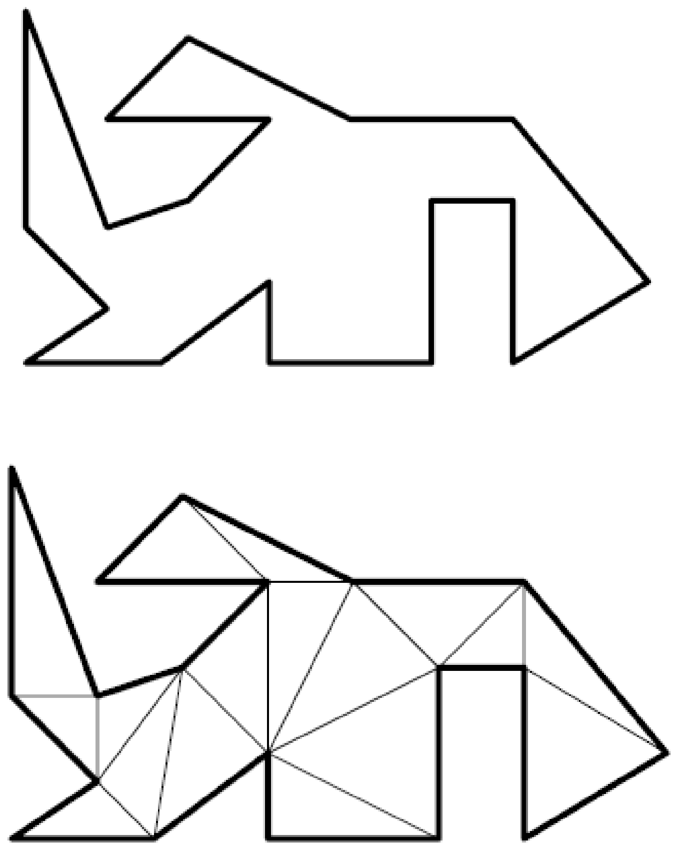
Art gallery, cont'd

- Four cameras cover this art gallery

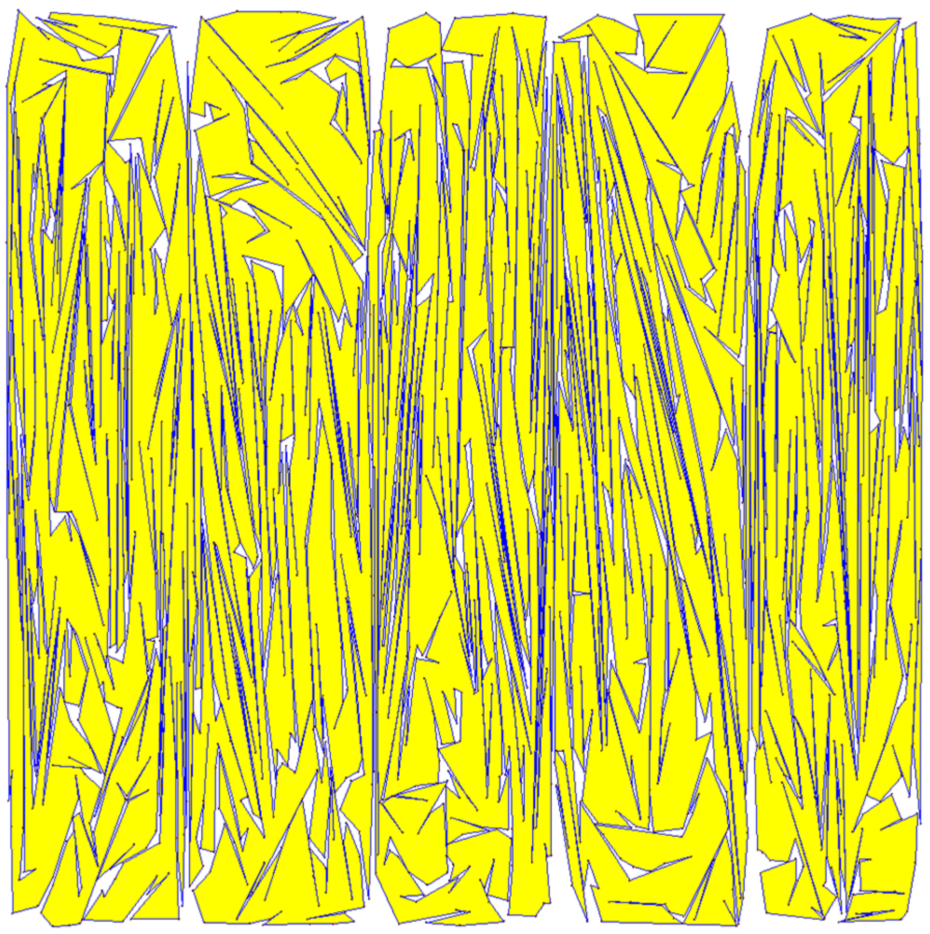


[Wikipedia:art gallery problem]

Art gallery and polygon triangulation, cont'd

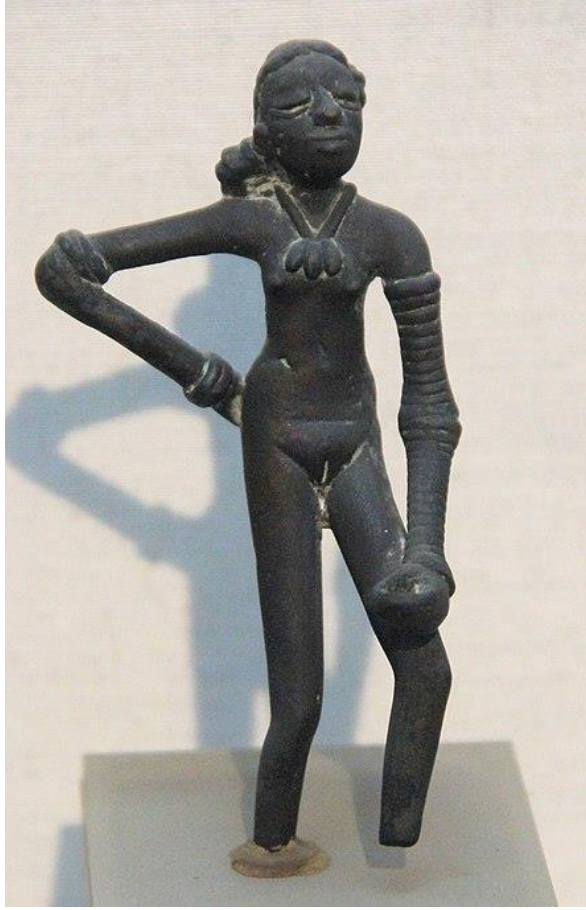


[CGAA]



[CG optimization competition, GFH]

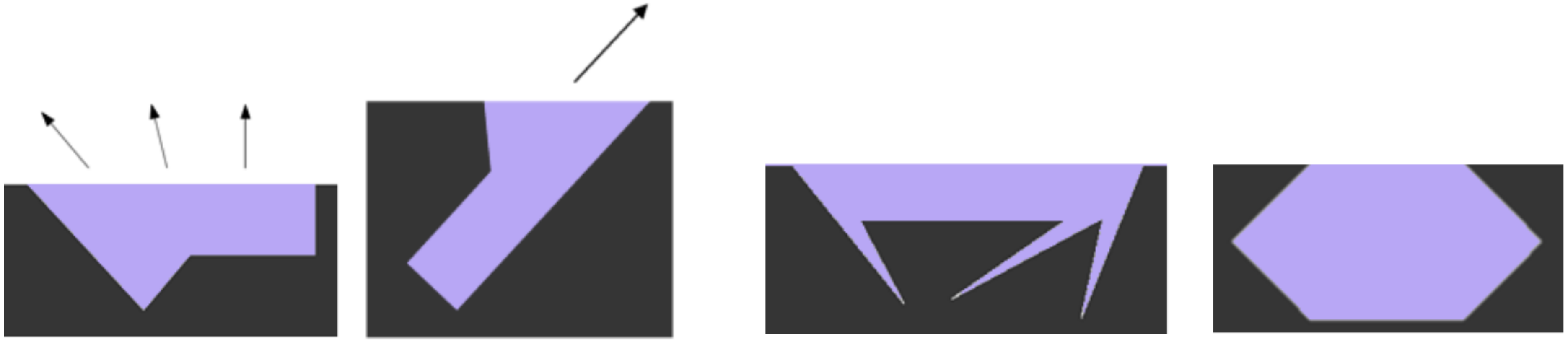
Casting and linear programming



[wikipedia:casting]

Casting and linear programming, cont'd

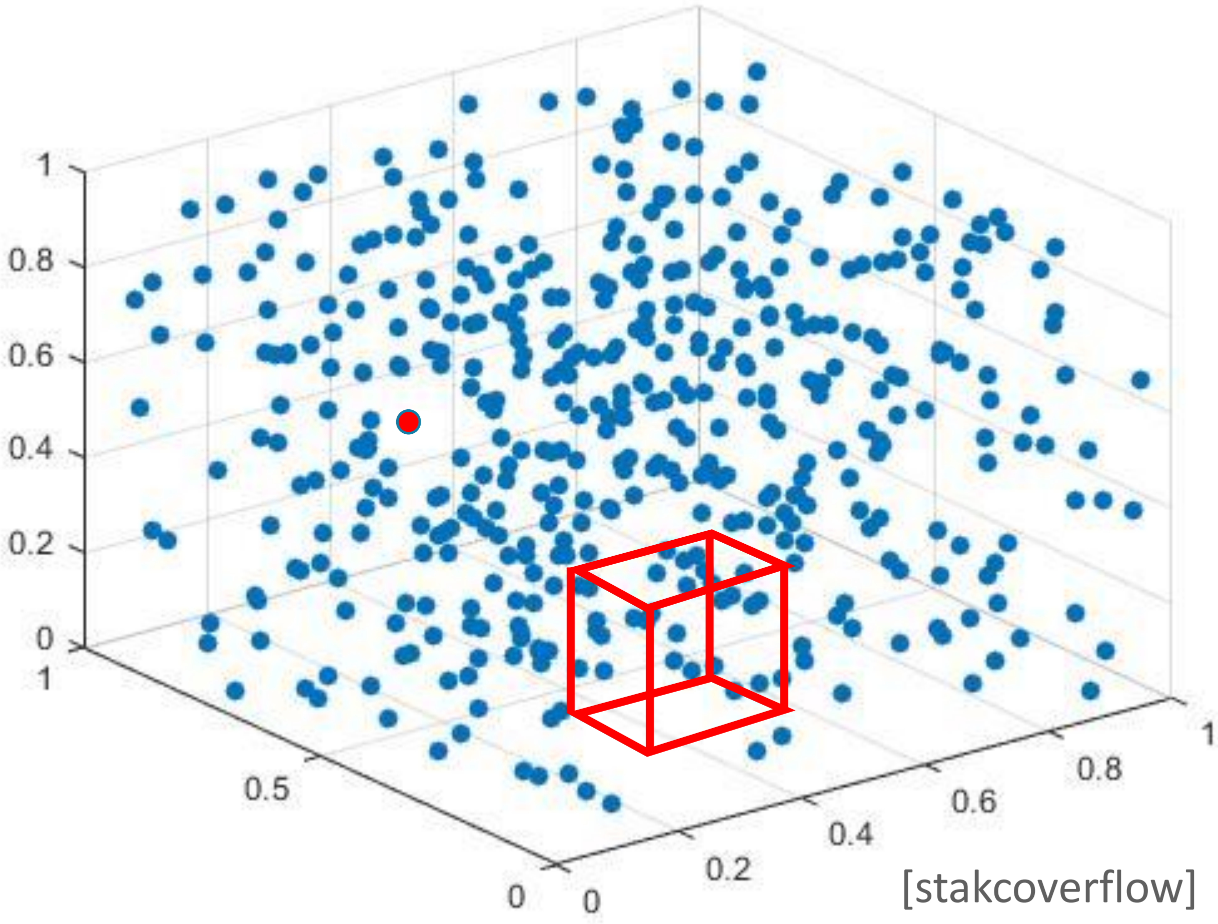
- Can a cast object (polyhedron) be taken out of its mold without breaking the mold?



- Intersection of half-spaces
- Linear programming

Orthogonal range search and nearest-neighbor search

- Nearest-neighbor search
- Orthogonal range search



[stakcoverflow]

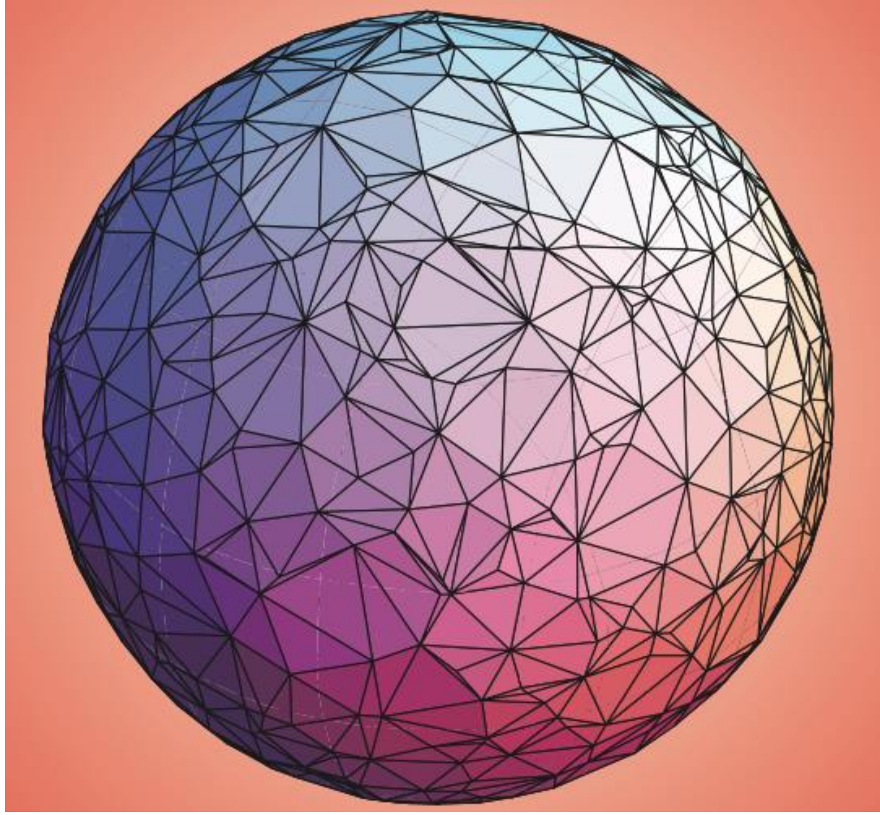
and more …

- Voronoi diagrams
- Delaunay triangulations
- Smallest enclosing disc
- Point location

Convex hull in 3D

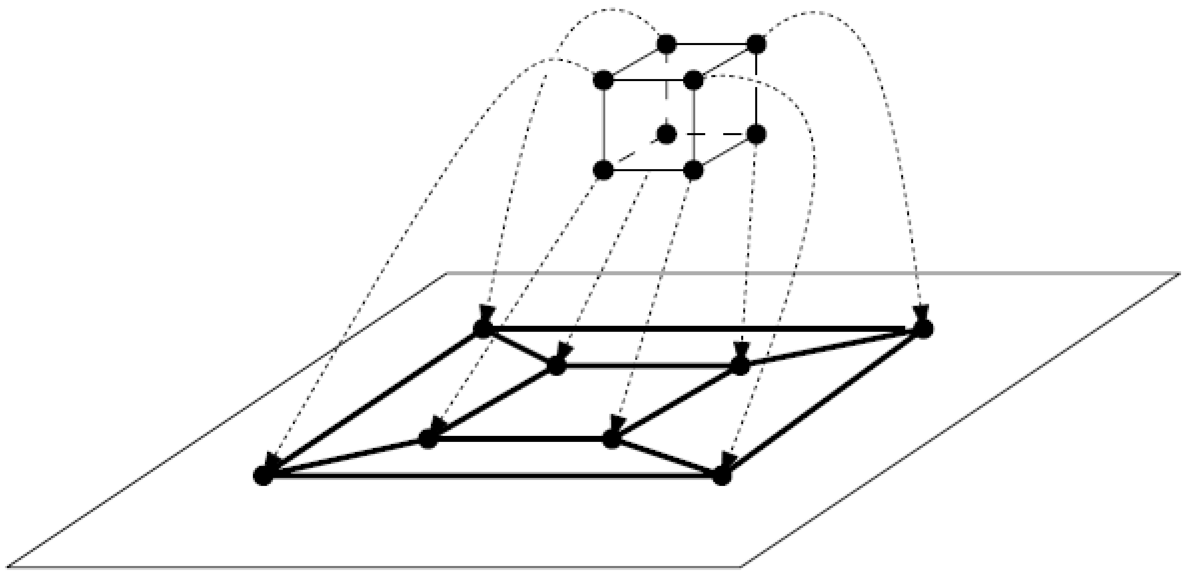
Convex hull in 3D

- the convex hull of a set P of n points in R^3 is a convex polytope whose vertices are points in P
- it therefore has at most n vertices
- its vertices and edges constitute a planar graph
- $CH(P)$ has at most $2n - 4$ faces and at most $3n - 6$ edges



[O'Rourke]

Convex polytopes and planar graphs



- the complexity bounds hold also for non-convex polytopes of *genus* zero with n vertices

THE END