Multi-Robot Motion Planning

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TAU

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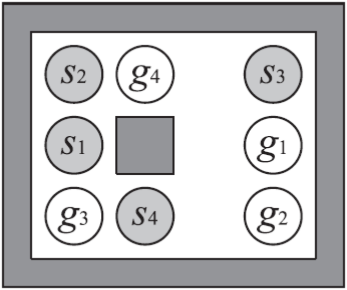

Main Approaches

Decoupled VS Centralized

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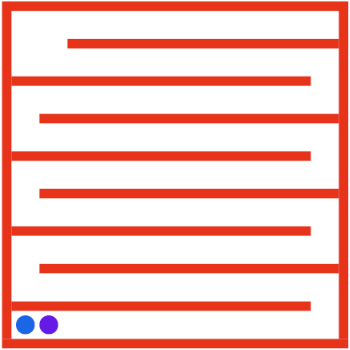

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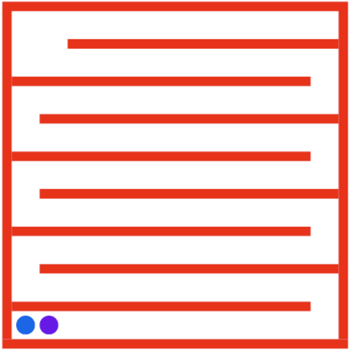

The two robots have to exchange positions.

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- Known techniques applicable only for small-scale problems with low degrees of freedom.

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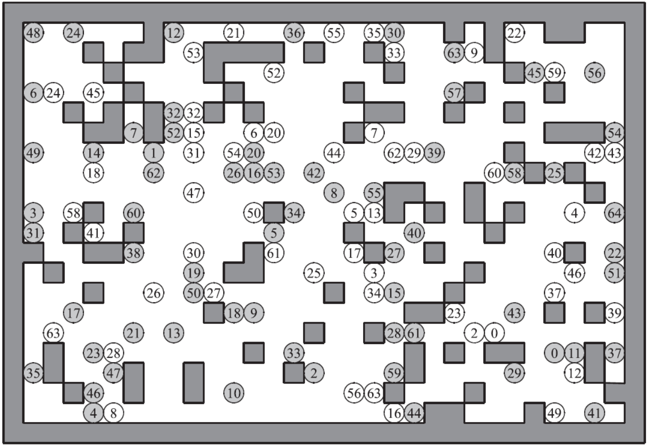

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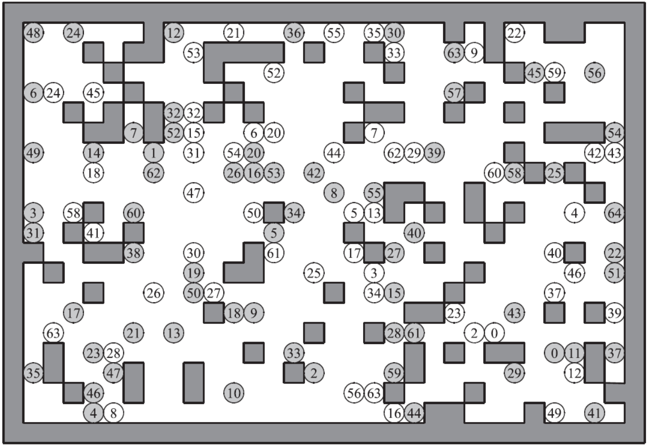

Introduction

Centralized Path Planning for Multiple Robots: Optimal Decoupling into Sequential Plans

Jur van den Berg   Jack Snoeyink   Ming Lin   Dinesh Manocha

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- In these sub-problems, individual robots will be coupled and considered as composite robots.

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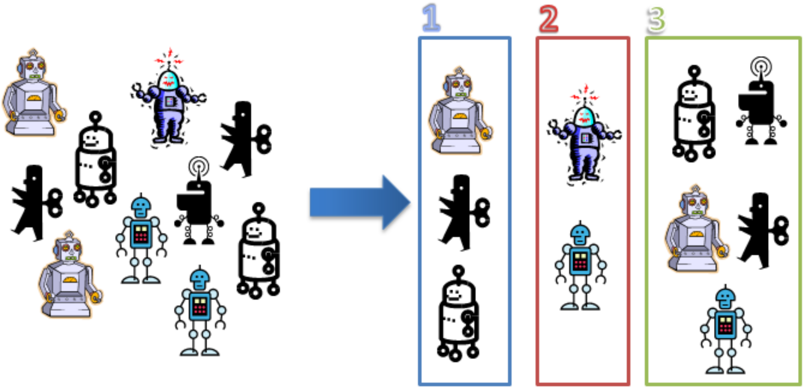

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- $C(r_i)$: Configuration space of robot r_i.
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Goal

Compute a path $\pi : [0,1] \to C(r_1) \times \ldots \times C(r_n)$ such that initially $\pi(0) = (s_1, \ldots, s_n)$, finally $\pi(1) = (g_1, \ldots, g_n)$, and for every $t \in (0,1)$ $\pi(t)$ describes a collision free (obstacles, robots) position.

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For a robot r_i the *Active Interval* τ_i is defined as the open interval from the first time r_i leaves its start position to the last time it reaches its goal position.

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Two robots r_i, r_j are coupled if their active intervals intersect, $\tau_i \cap \tau_j \neq \emptyset$.

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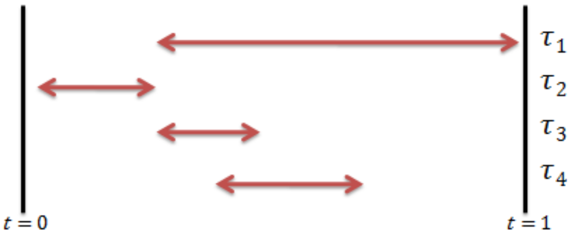

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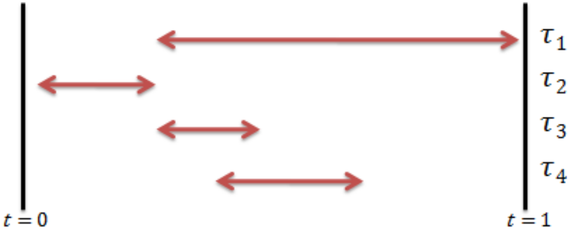

The coupled robots are $\{r_1, r_3\}, \{r_1, r_4\}, \{r_3, r_4\}$.

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A *Composite Robot* R is a subset of $\{r_1, \ldots, r_n\}$.

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Definition

Execution Sequence is an ordered partition of the n robots into a sequence $S = (R_1, \ldots, R_k)$ of composite robots, such that

- $R_1 \cup \ldots \cup R_k = \{r_1, \ldots, r_n\}$
- For every $i \neq j$ $R_i \cap R_j = \emptyset$.

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Definition

S is *optimal* if it is the solution sequence with the lowest degree of its largest *composite robot*.

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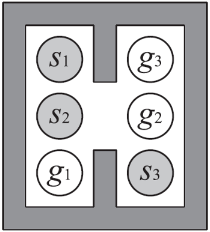

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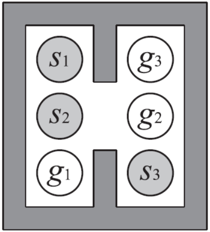

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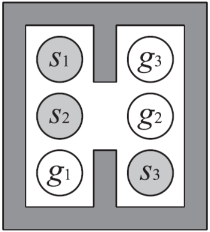

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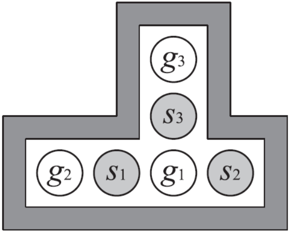

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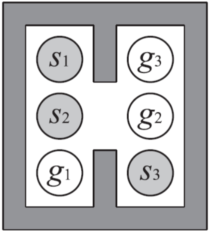

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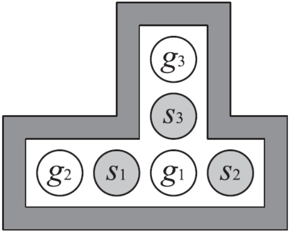

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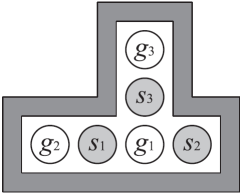
For example, $S = [r_3, r_1 r_2]$ is valid for the scene



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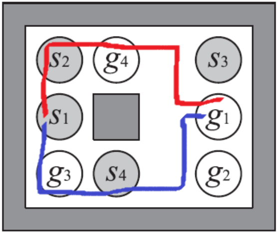

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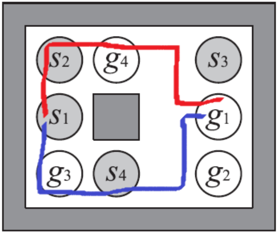

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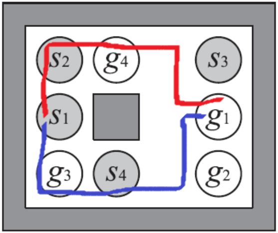

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- What will be the order of r_1, r_3, r_4 in the solution sequence?

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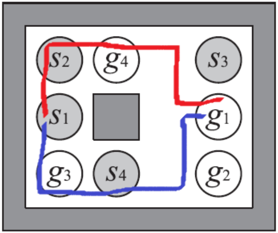

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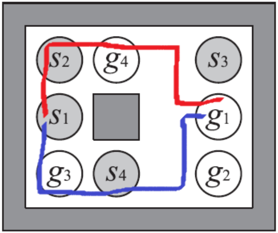

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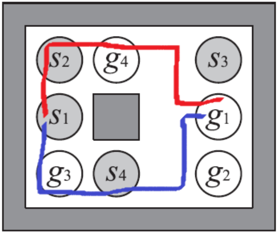

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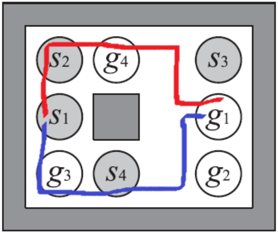

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And combined:

$$P(r_1) = (r_2 \prec r_1 \wedge r_1 \prec r_4) \vee (r_1 \prec r_3 \wedge r_4 \prec r_1)$$

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An execution sequence $S = (R_1, \ldots, R_k)$ *is a* solution sequence *iff* S *satisfies the constraints expression* $P(R_1) \wedge \ldots \wedge P(R_k)$.

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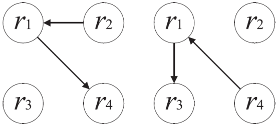

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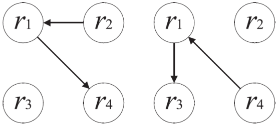

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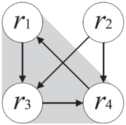

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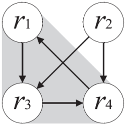

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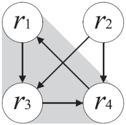

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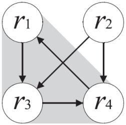

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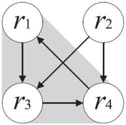

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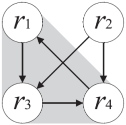

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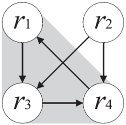

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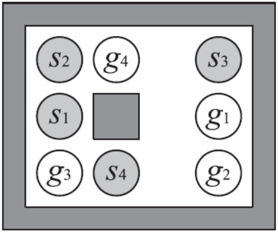
- We stop when there exists $J \in E$ for which all robots in $S(J)$ have been planned.

Incrementally Building the Execution Sequence: Example

We will run the algorithm on the problem:



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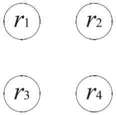

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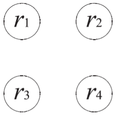

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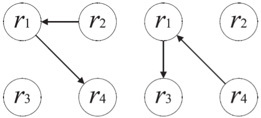
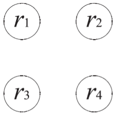


Iteration 2. $L = \{r_1\}$, $R_{\min} = r_2$, $\mathcal{P}(r_2) = r_2 \prec r_4 \vee (r_1 \prec r_2 \wedge r_2 \prec r_3 \wedge r_4 \prec r_2)$.

Incrementally Building the Execution Sequence: Example



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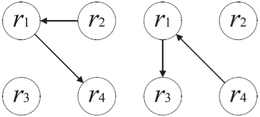

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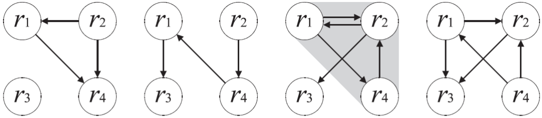
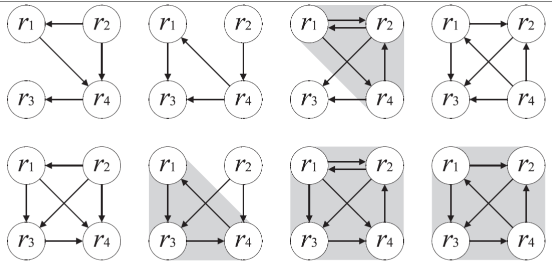


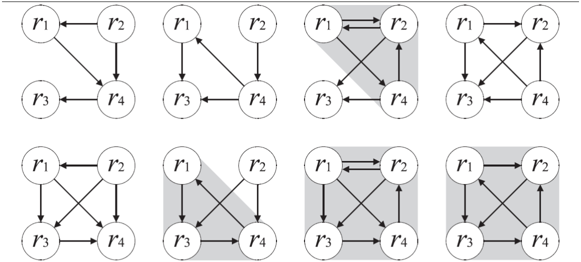
Iteration 3. $L = \{r_1, r_2\}$, $R_{\min} = r_3$, $\mathcal{P}(r_3) = r_4 \prec r_3 \vee (r_3 \prec r_4 \wedge r_2 \prec r_3 \wedge r_1 \prec r_3)$.

Incrementally Building the Execution Sequence: Example



Iteration 4. $L = \{r_1, r_2, r_3\}$, $R_{\min} = r_4$, $\mathcal{P}(r_4) = \top$, so no change to graphs.

Incrementally Building the Execution Sequence: Example



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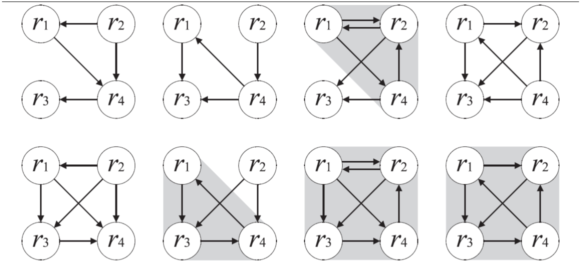

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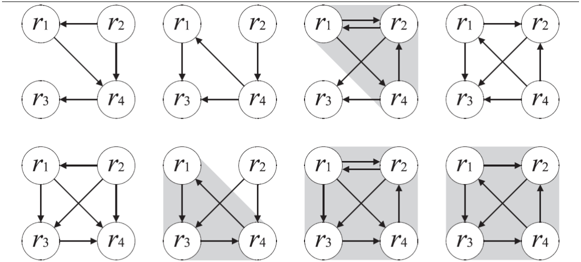

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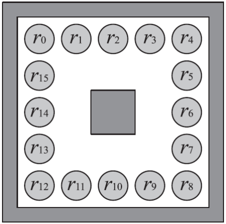
- After the 4th iteration, $L = \{r_1, r_2, r_3, r_4\}$.
- J_1, J_2, J_4, J_5 have all their SCC in L.
- The algorithm terminates and returns $S(J_1)$.

Scene 1



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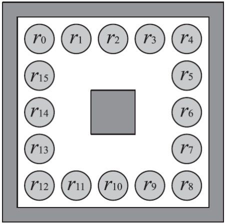

- Each robot r_i changes position with $r_{((i+8) \mod 16)}$

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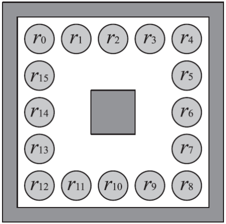
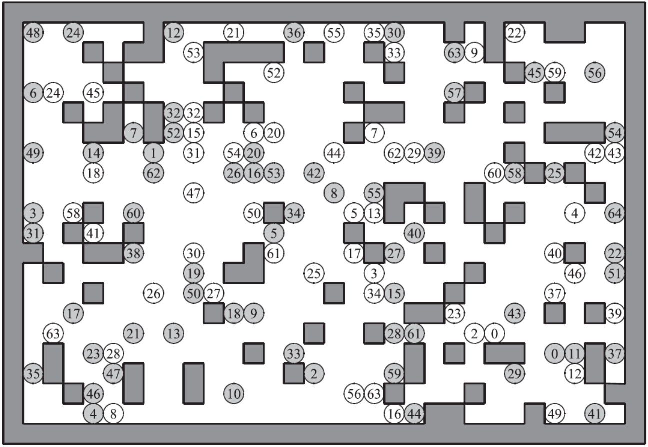


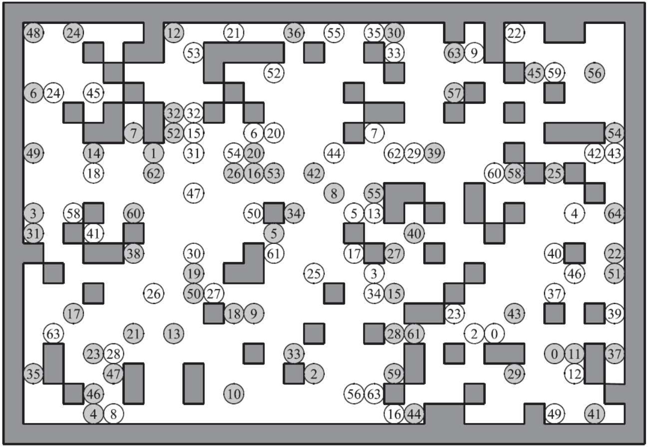
- Each robot r_i changes position with $r_{((i+8) \mod 16)}$
- The algorithm returned

$$(r_0 r_7 r_8 r_{15}, r_1 r_9, r_2 r_{10}, r_3 r_{11}, r_4 r_5 r_{12} r_{13}, r_6 r_{14}, r_7 r_{15})$$

Scene 2

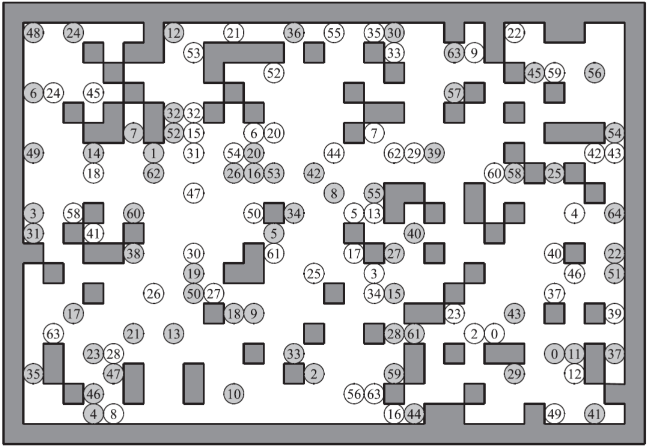


Scene 2



- 65 robots

Scene 2



- 65 robots
- Solution sequence involves only **individual** robots!

Project

- Two possible settings, two possible CR Planners:

Project

- Two possible settings, two possible CR Planners:
  - Grid workspace: motion planning on graphs.

Project

- Two possible settings, two possible CR Planners:
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  - Polygonal robots and obstacles (harder): sampling-based techniques.

Project

- Two possible settings, two possible CR Planners:
  - Grid workspace: motion planning on graphs.
  - Polygonal robots and obstacles (harder): sampling-based techniques.
- In both cases, the main algorithm remains the same.

The End

"... and the robots lived happily ever after."