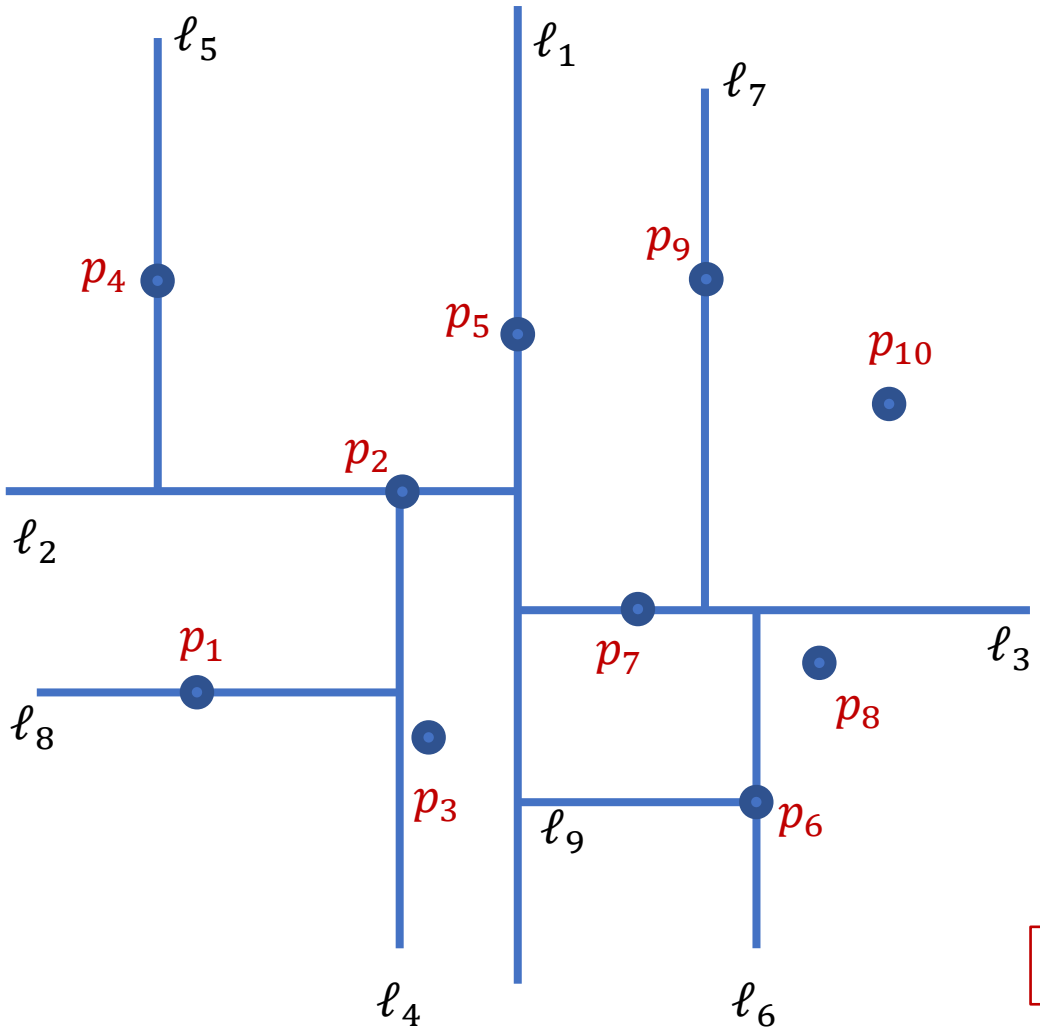
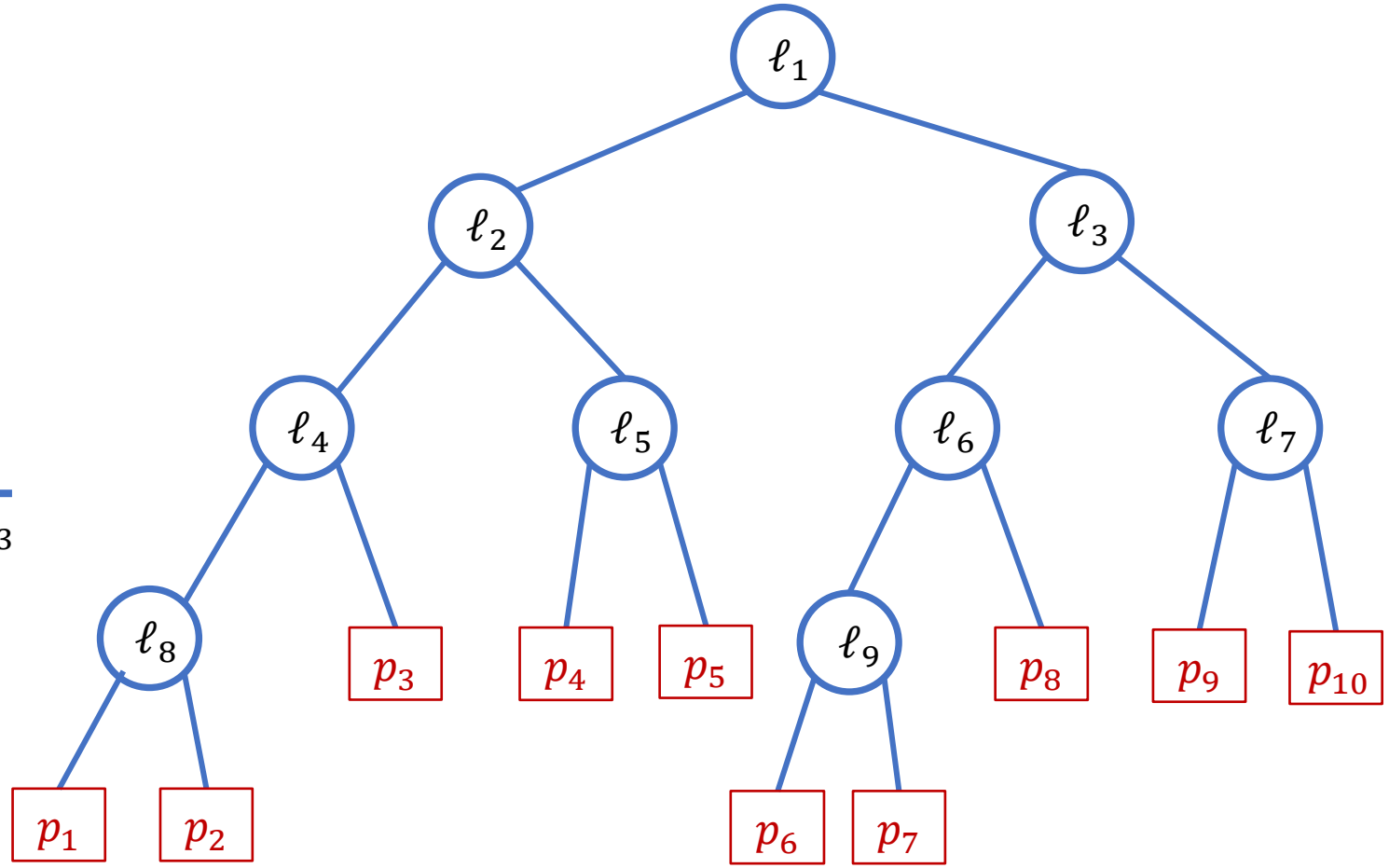
Nearest-neighbor search using Kd-trees

Kd-tree


ℓ_5
ℓ_1
ℓ_7
p_4
p_9
p_5
p_{10}
p_2
ℓ_2
p_1
p_7
ℓ_3
p_8
ℓ_8
p_3
ℓ_9
p_6
ℓ_4
ℓ_6


ℓ_1
ℓ_2
ℓ_3
ℓ_4
ℓ_5
ℓ_6
ℓ_7
ℓ_8
p_3
p_4
p_5
ℓ_9
p_8
p_9
p_{10}
p_1
p_2
p_6
p_7

The nearest-neighbor problem

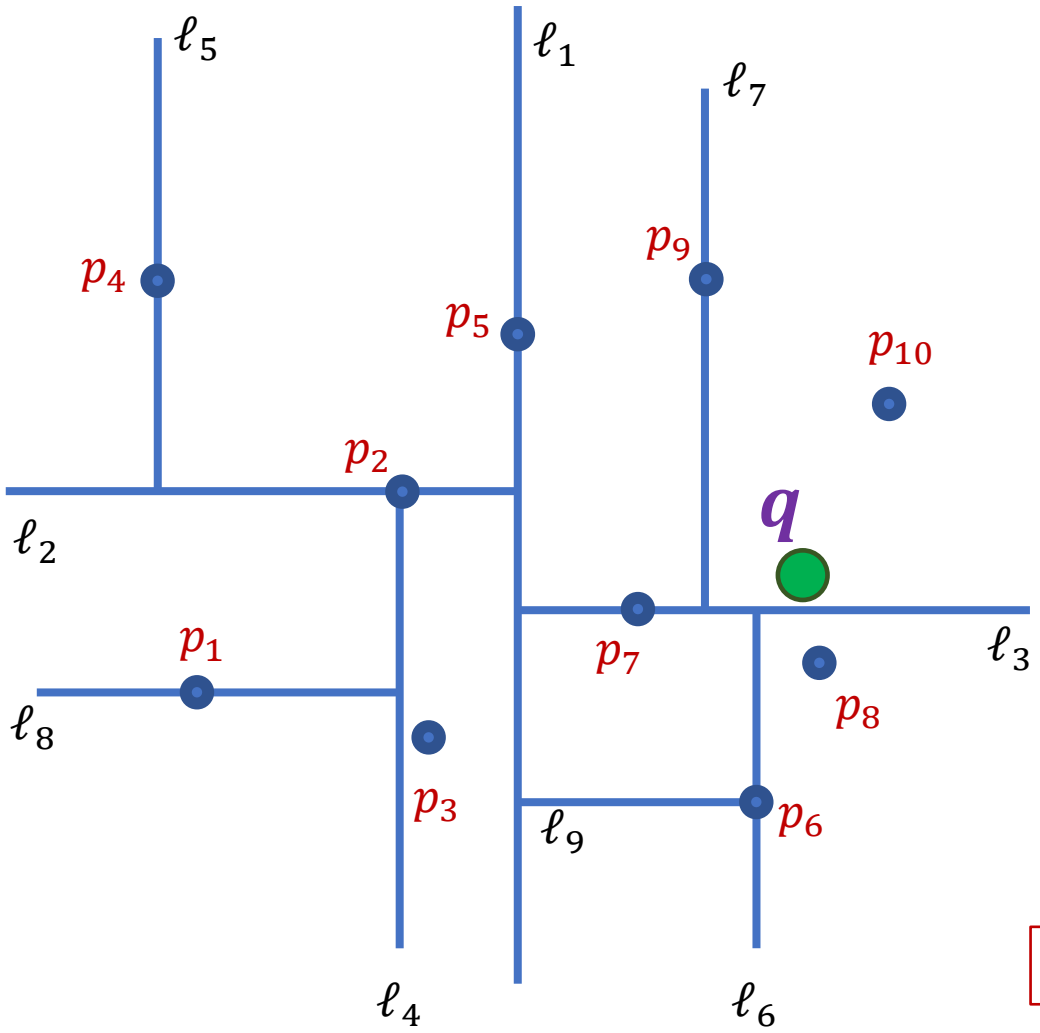
- Given a set of points P and a query point q, return the point $p \in P$ closest to q

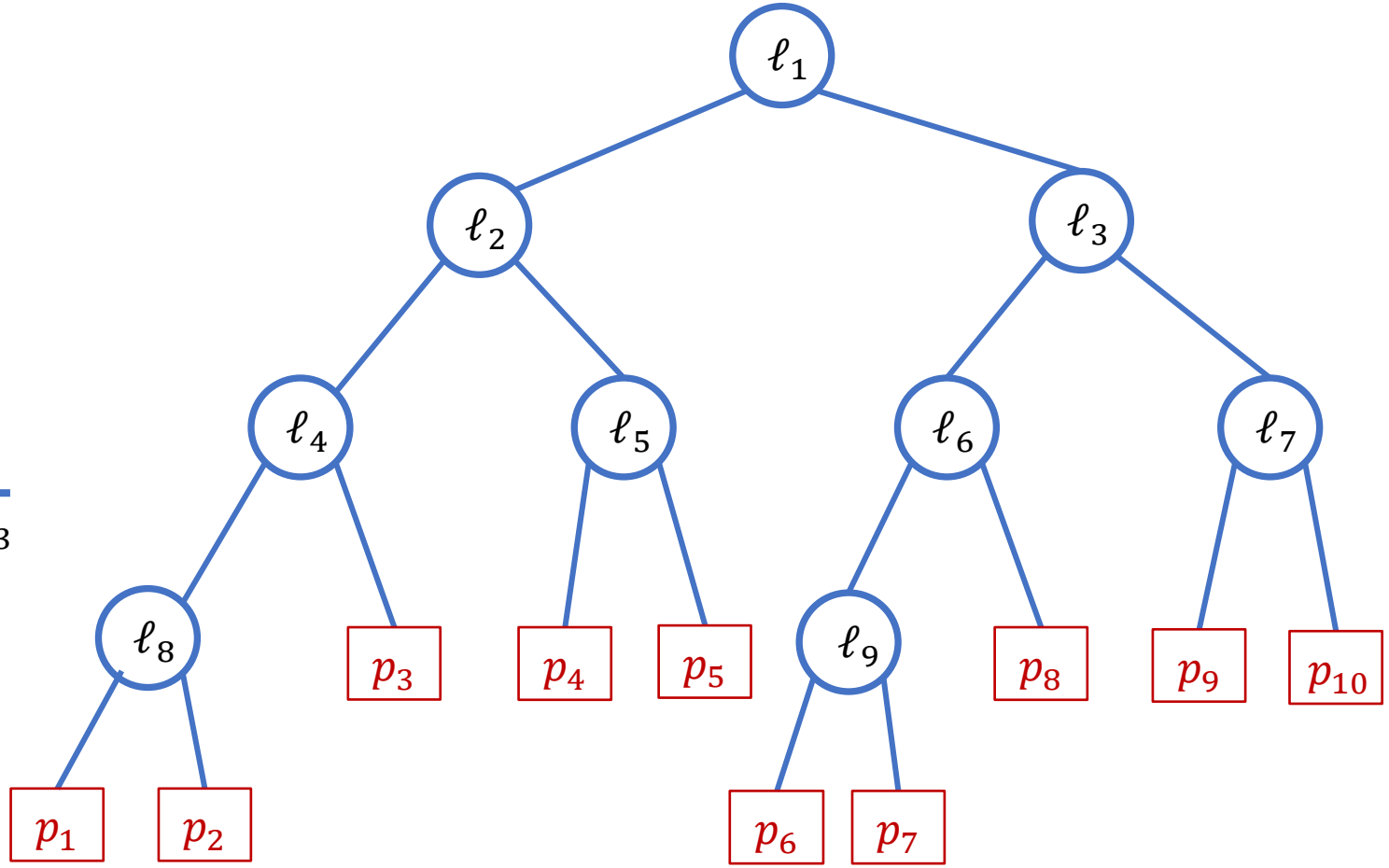
- Often, we would like to preprocess P into a data structure that efficiently answers such queries

Nearest-neighbor search using a Kd-tree

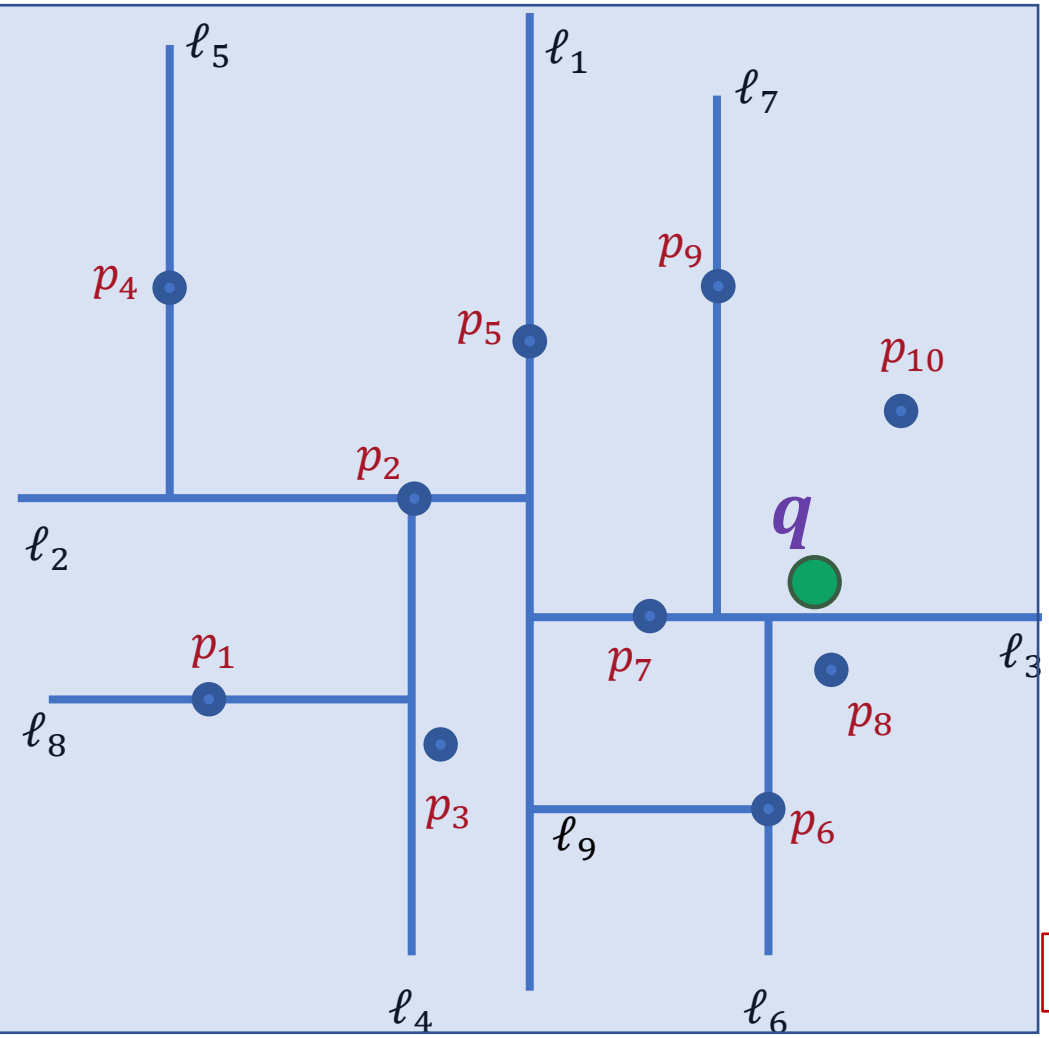
- Search the tree recursively to find the point (leaf) in the same cell as the query point q
- This is our candidate for the nearest neighbor of q
- On the way up search each subtree where a closer point than the current candidate might be found

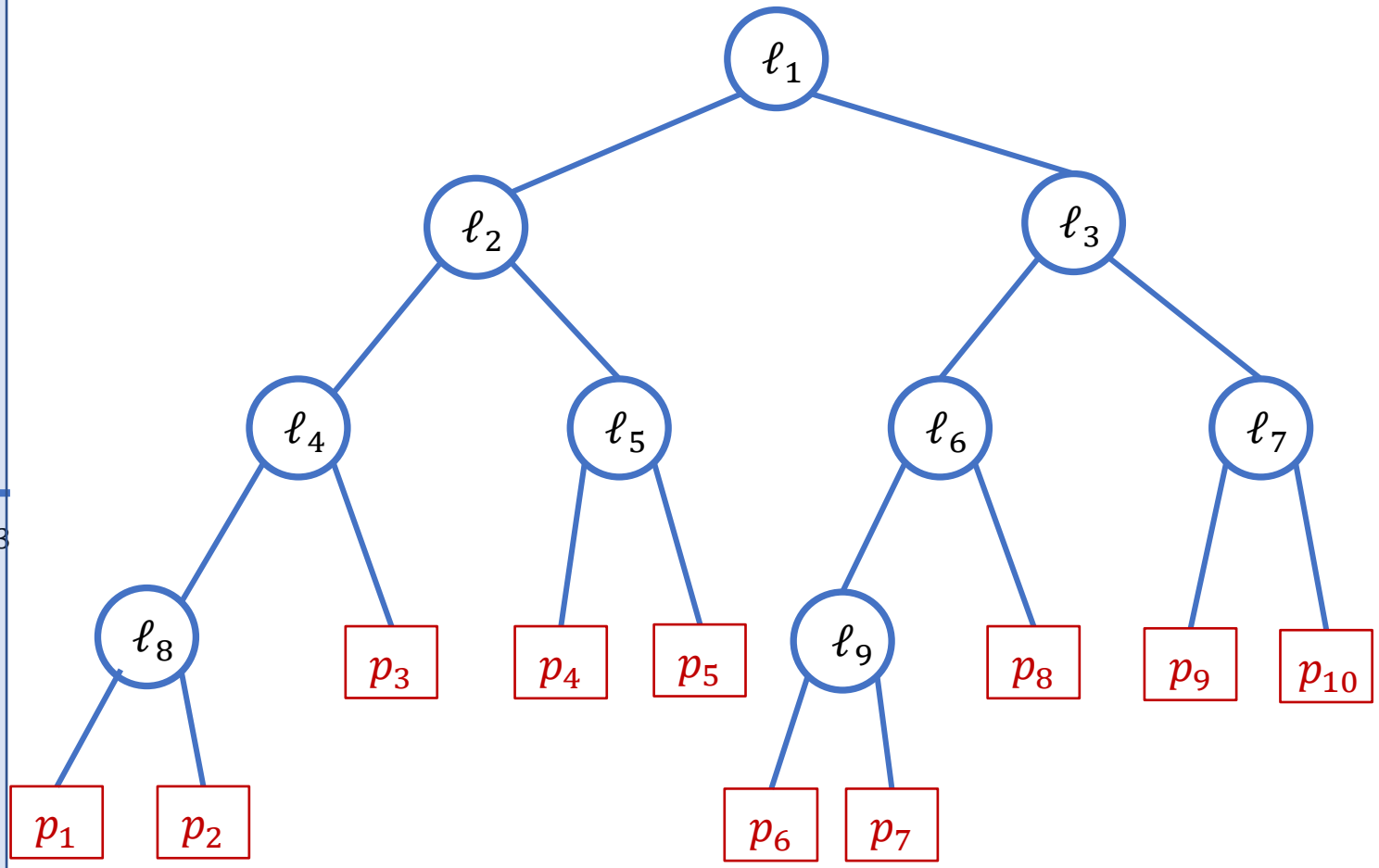
Nearest-neighbor search using Kd-tree





Step 1: find the cell of q





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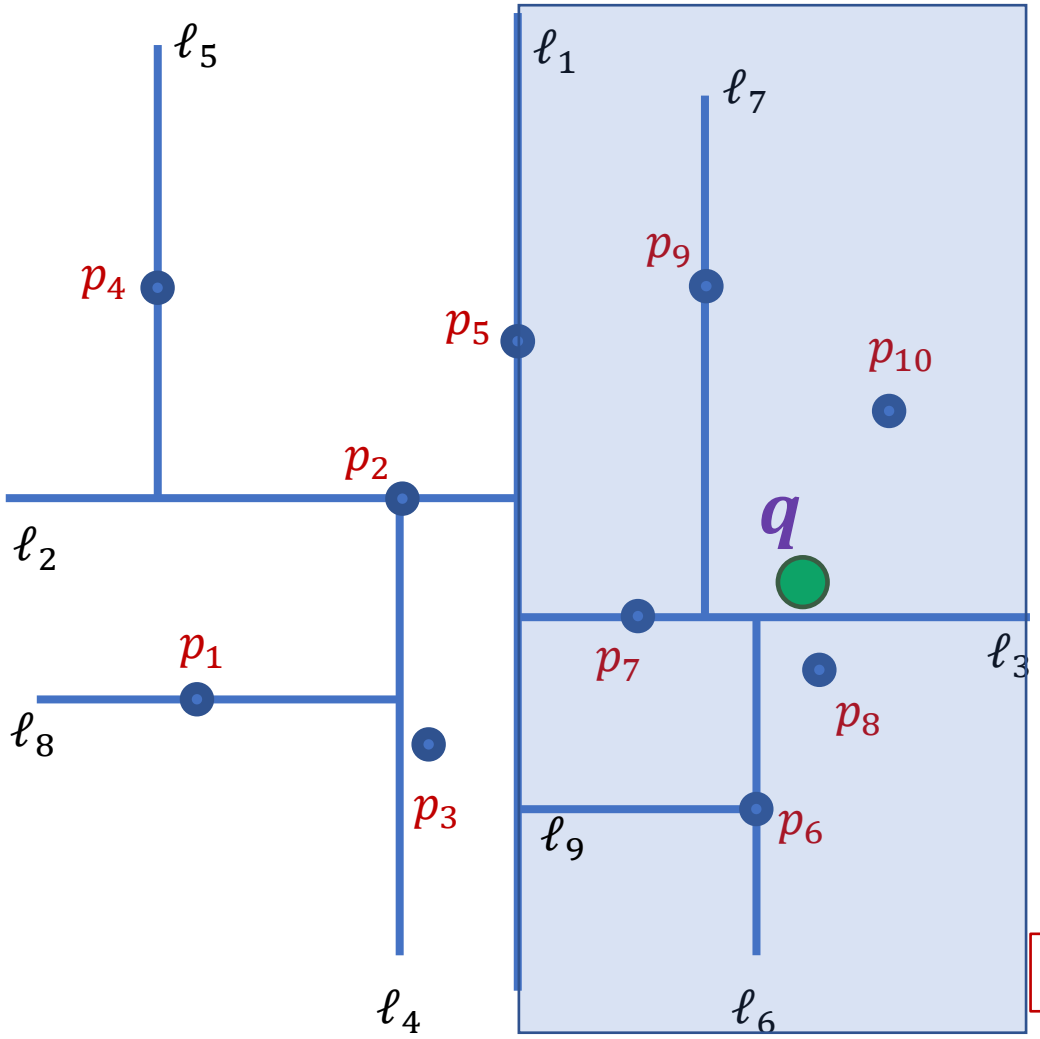

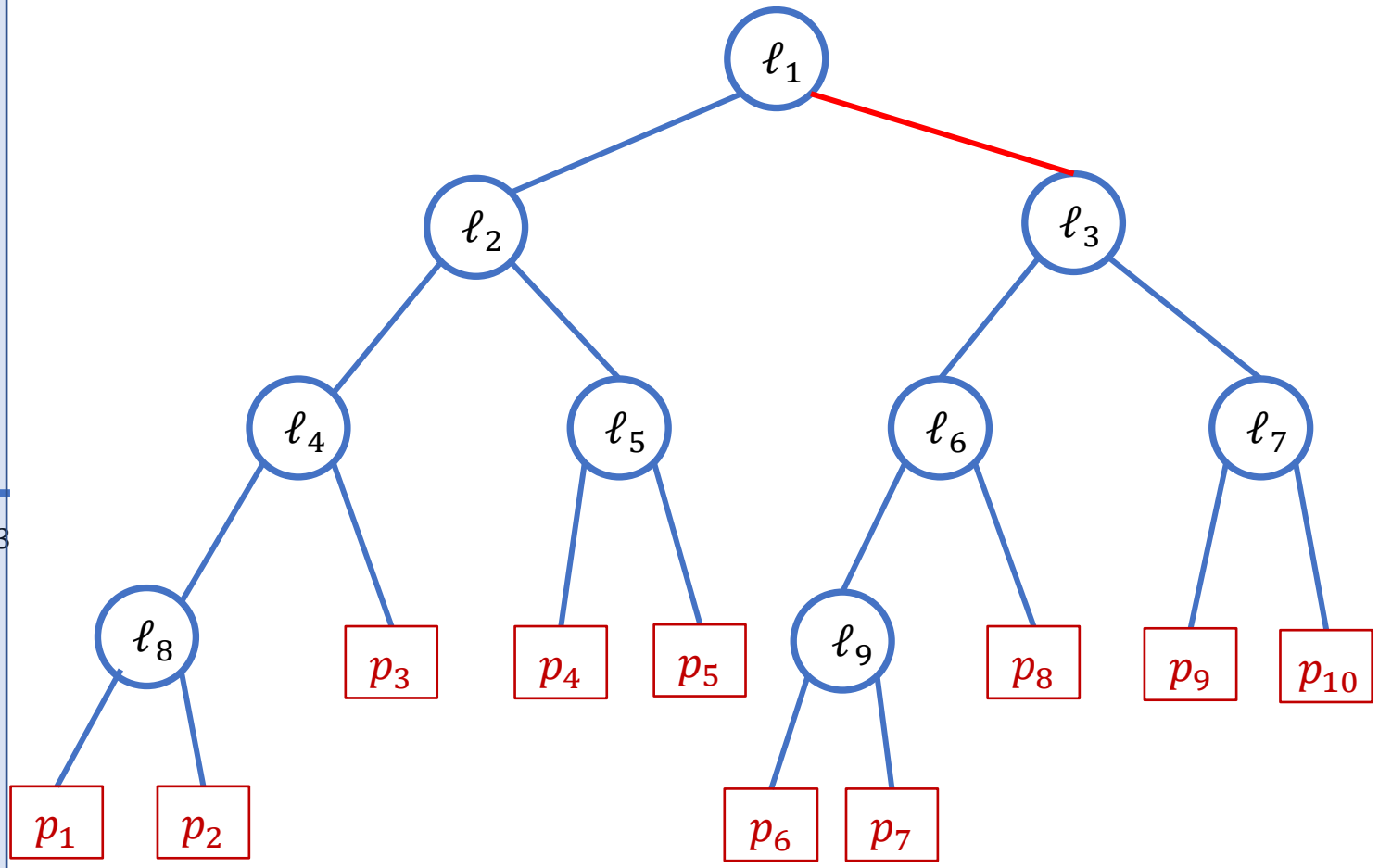

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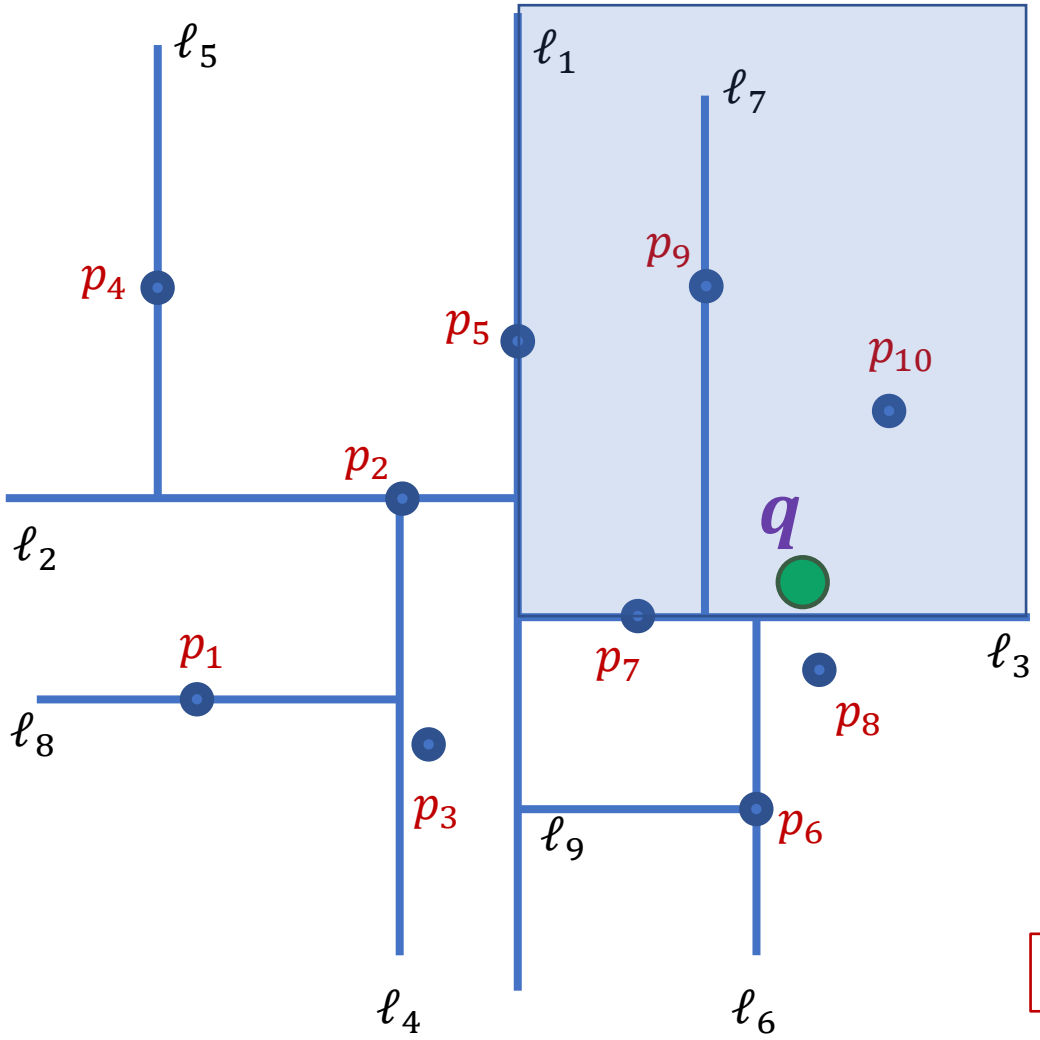
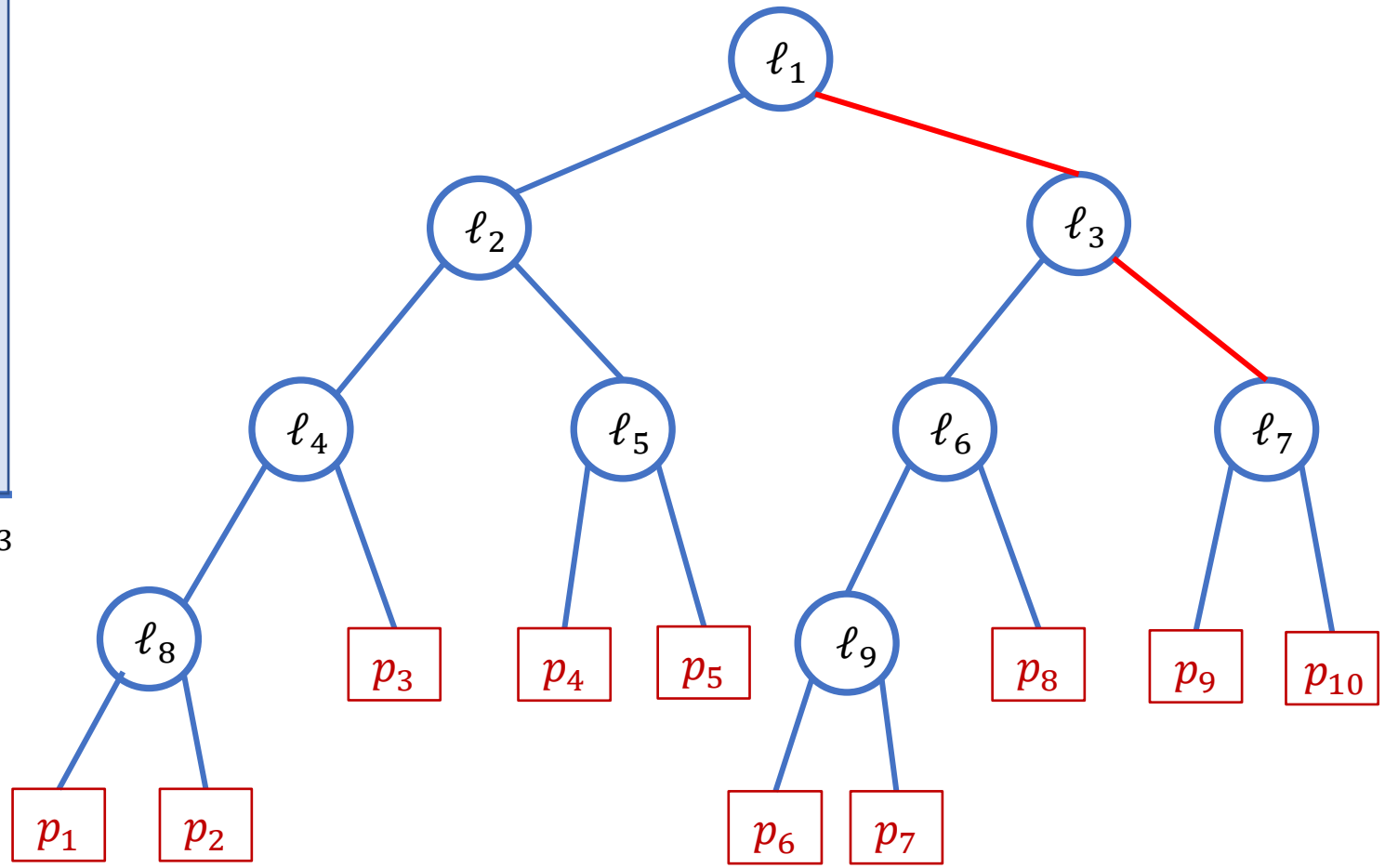
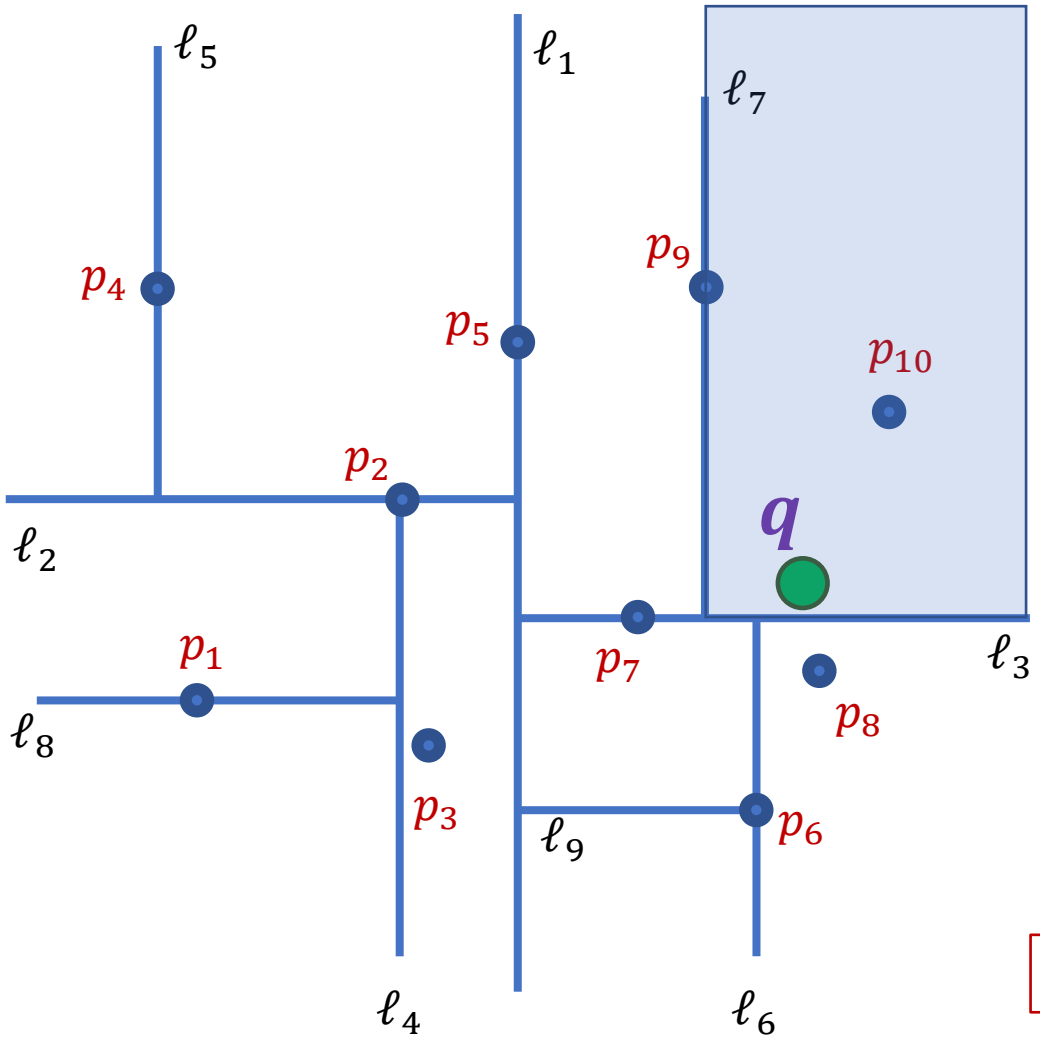
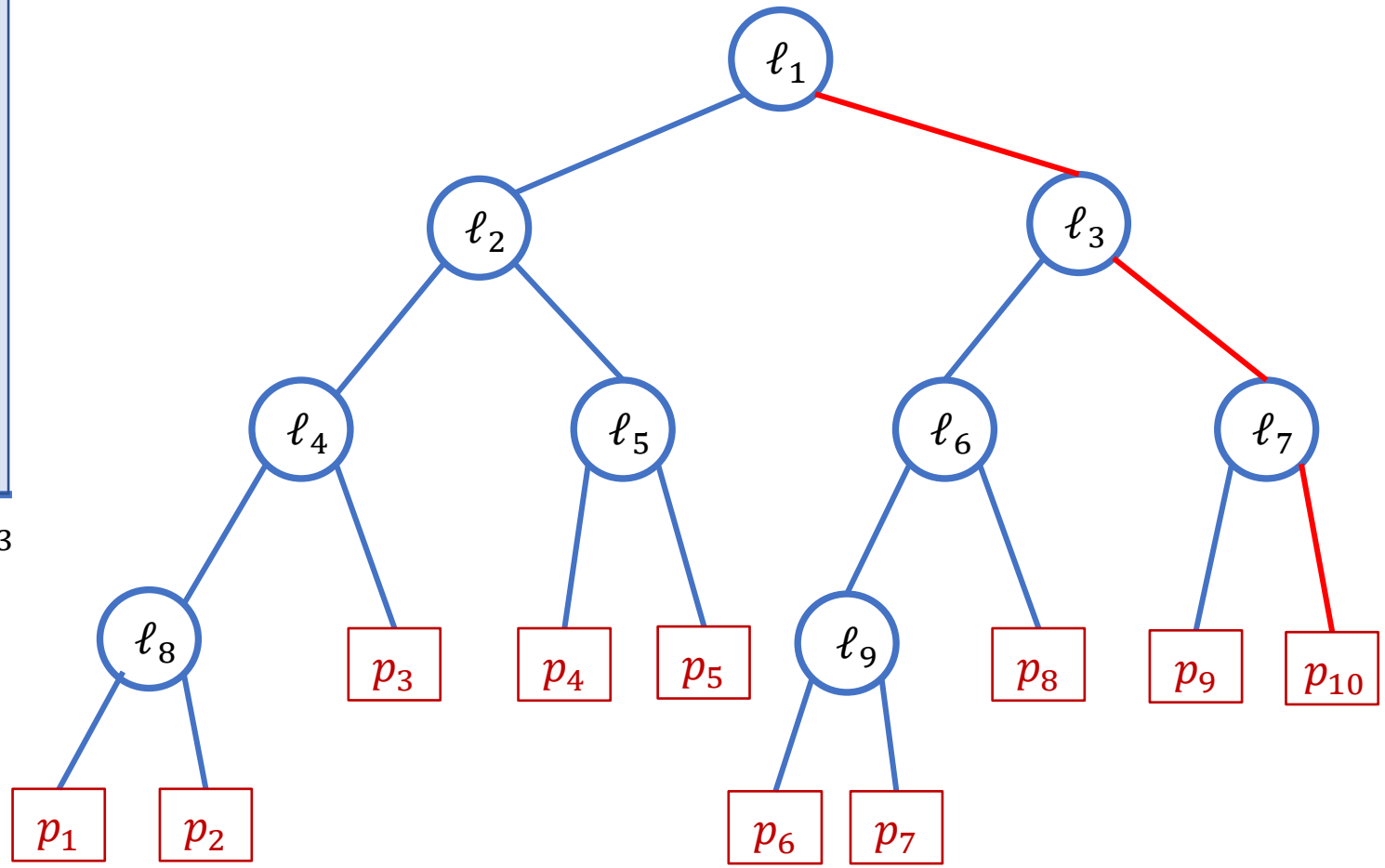


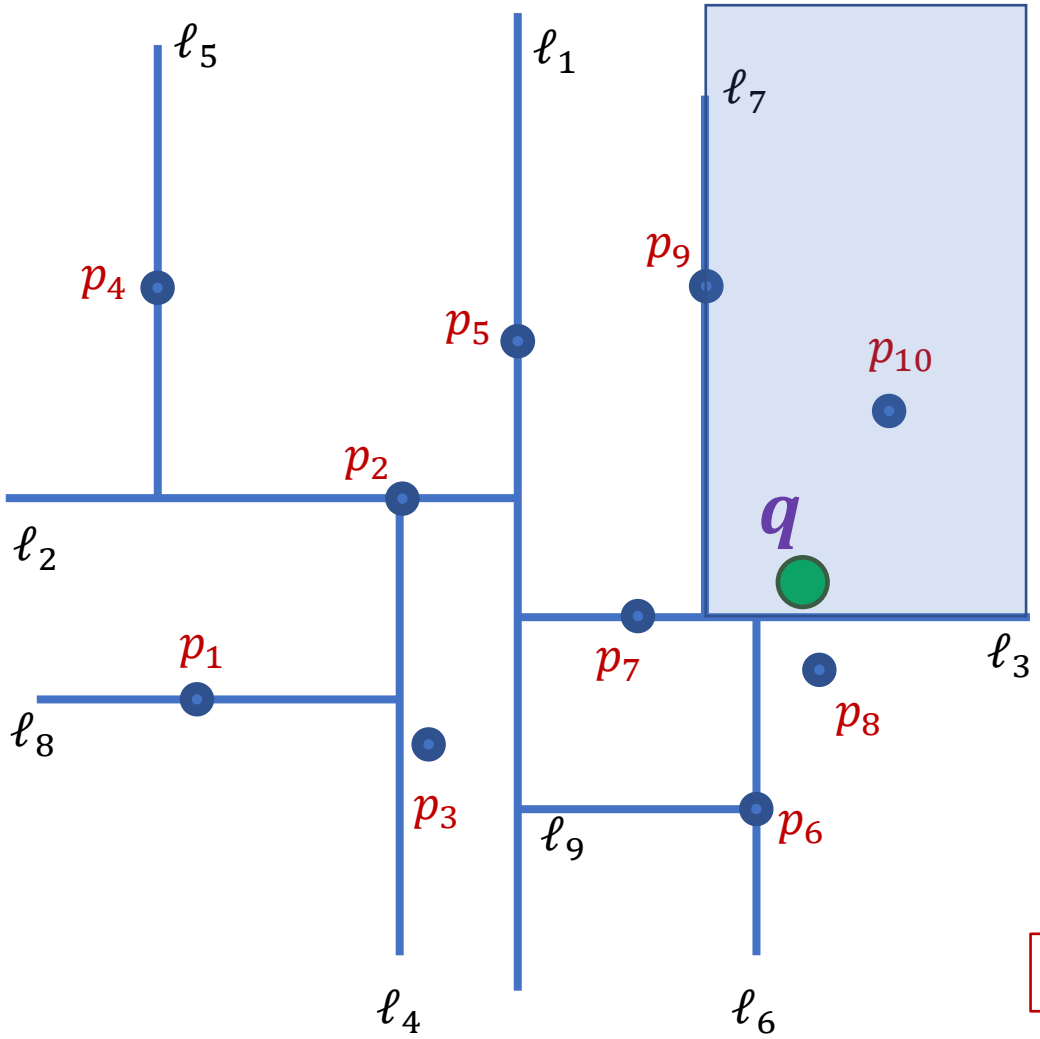


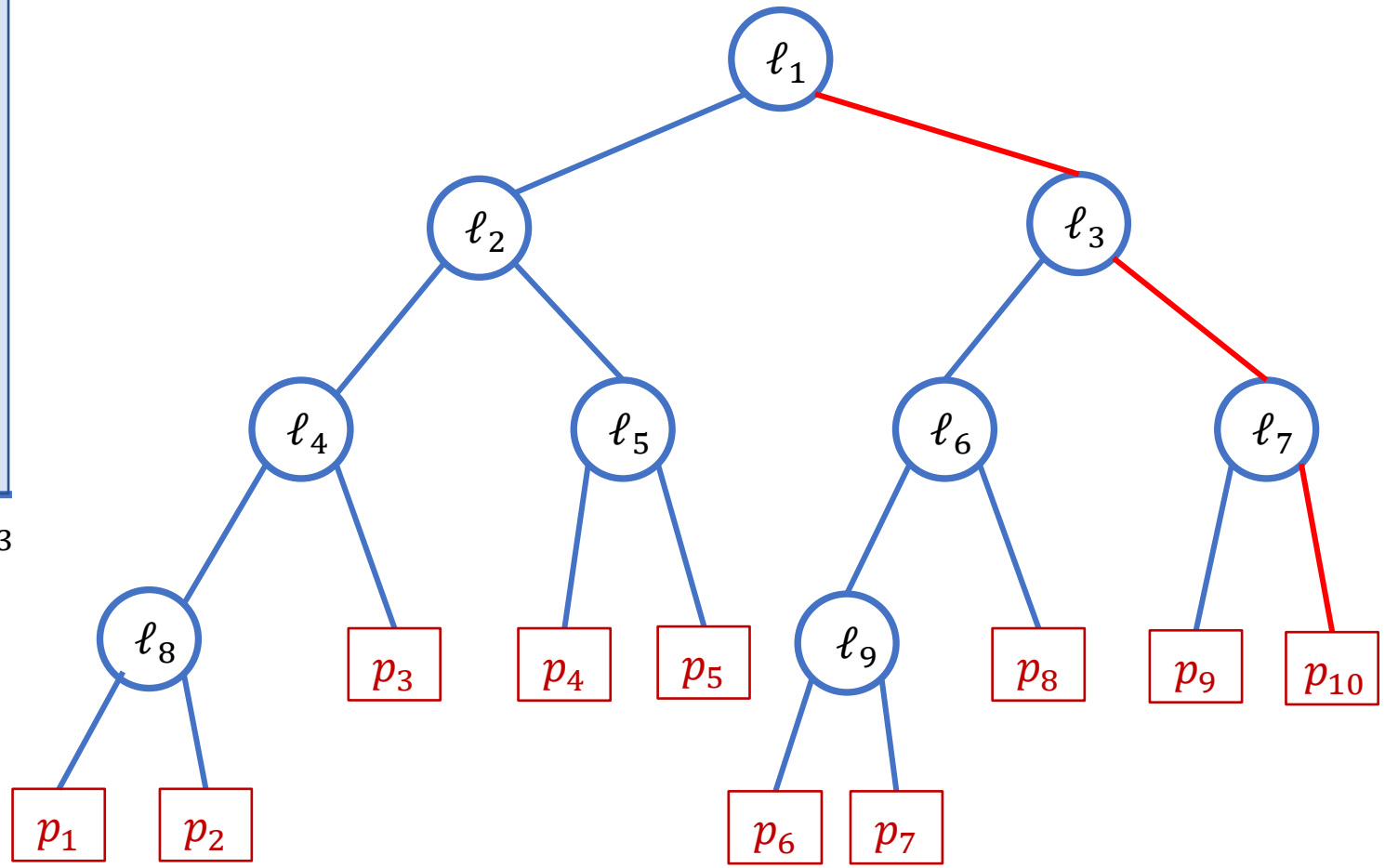
Step 1: find the cell of q



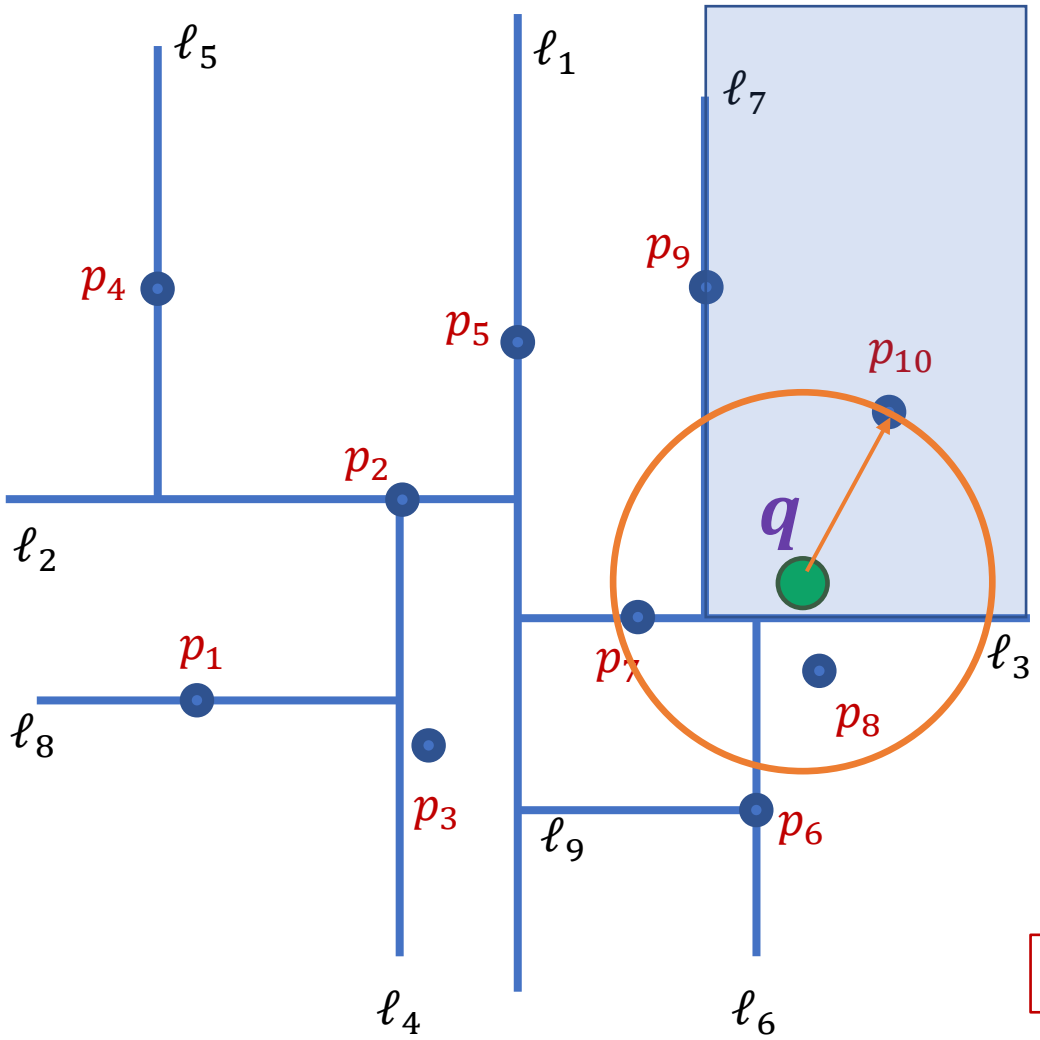


Current candidate: p_{10}





Step 2



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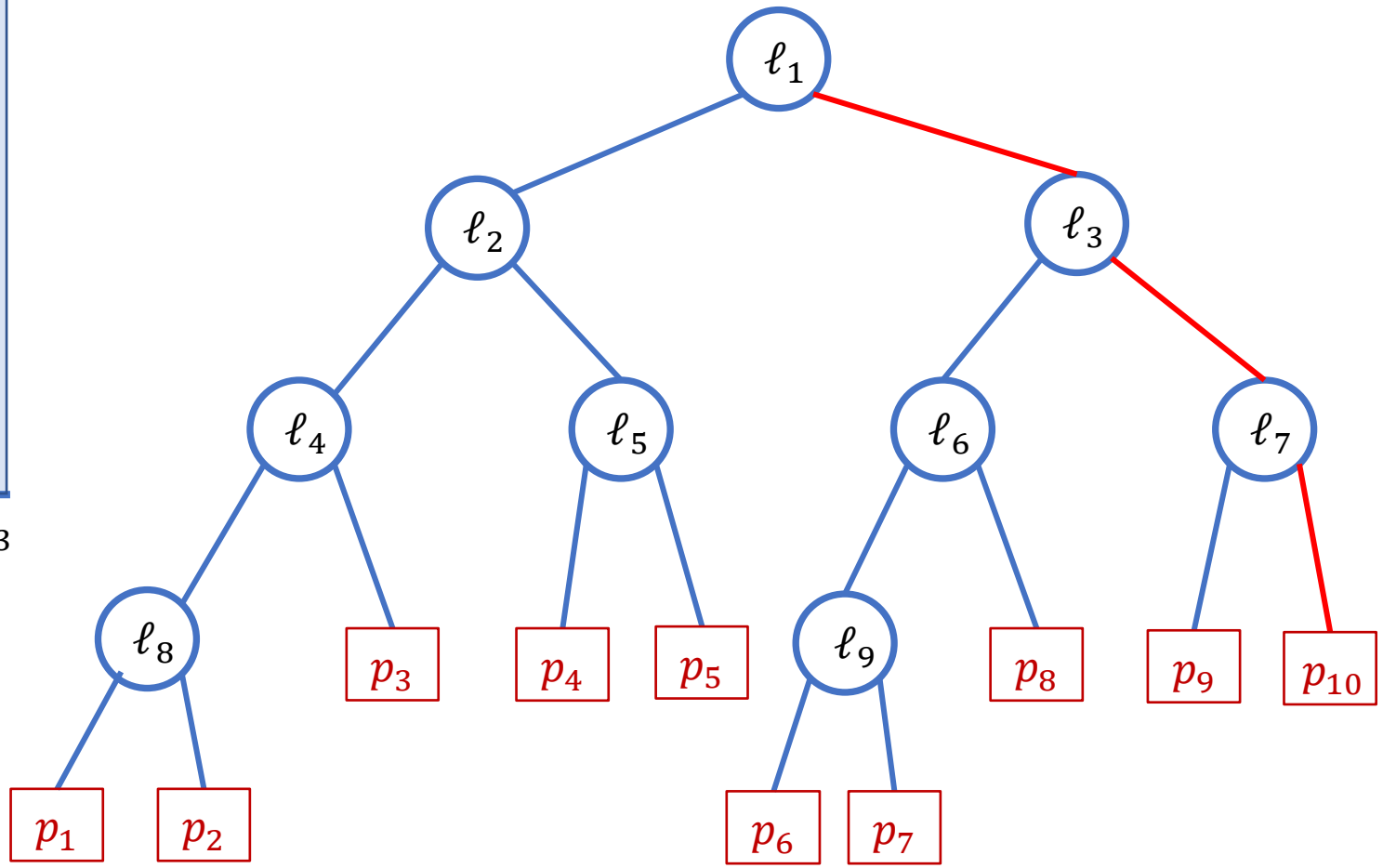

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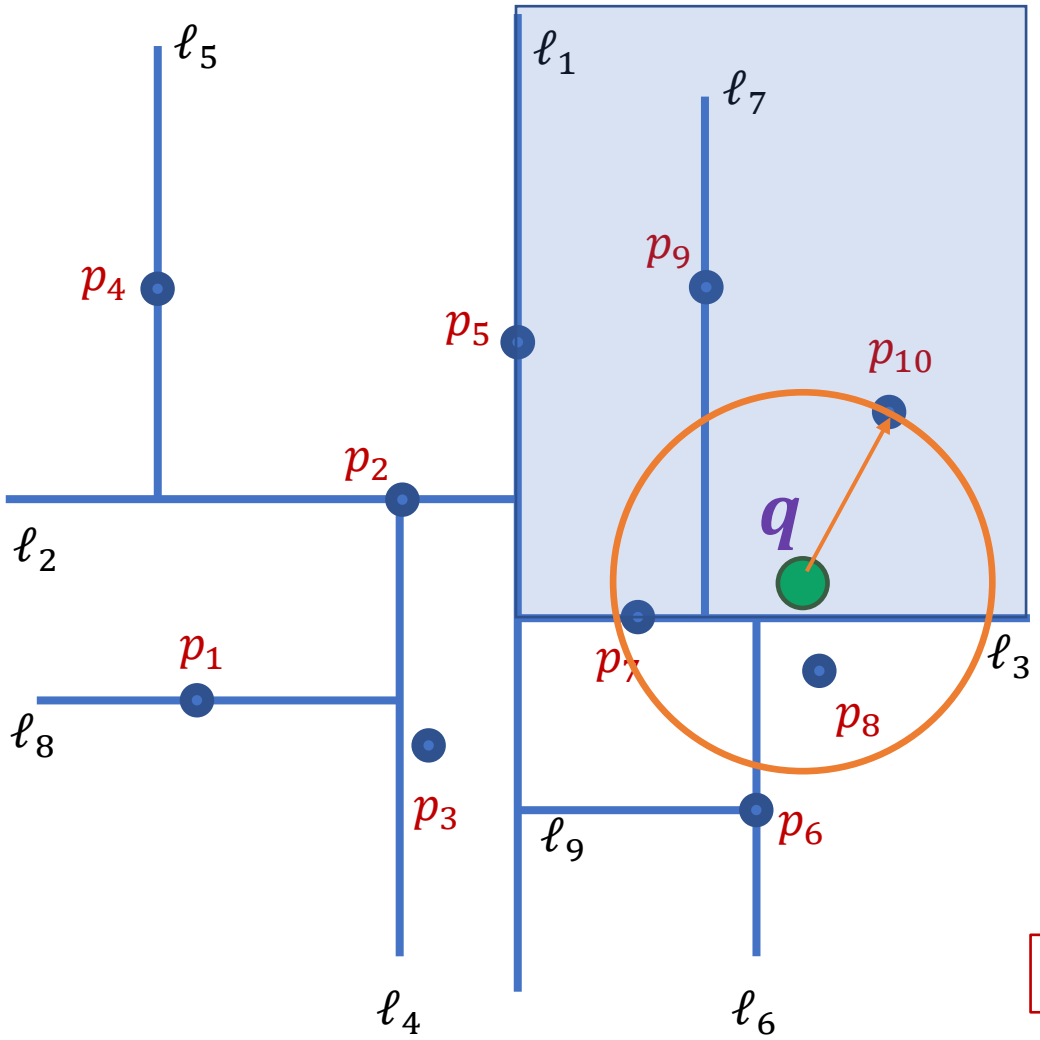

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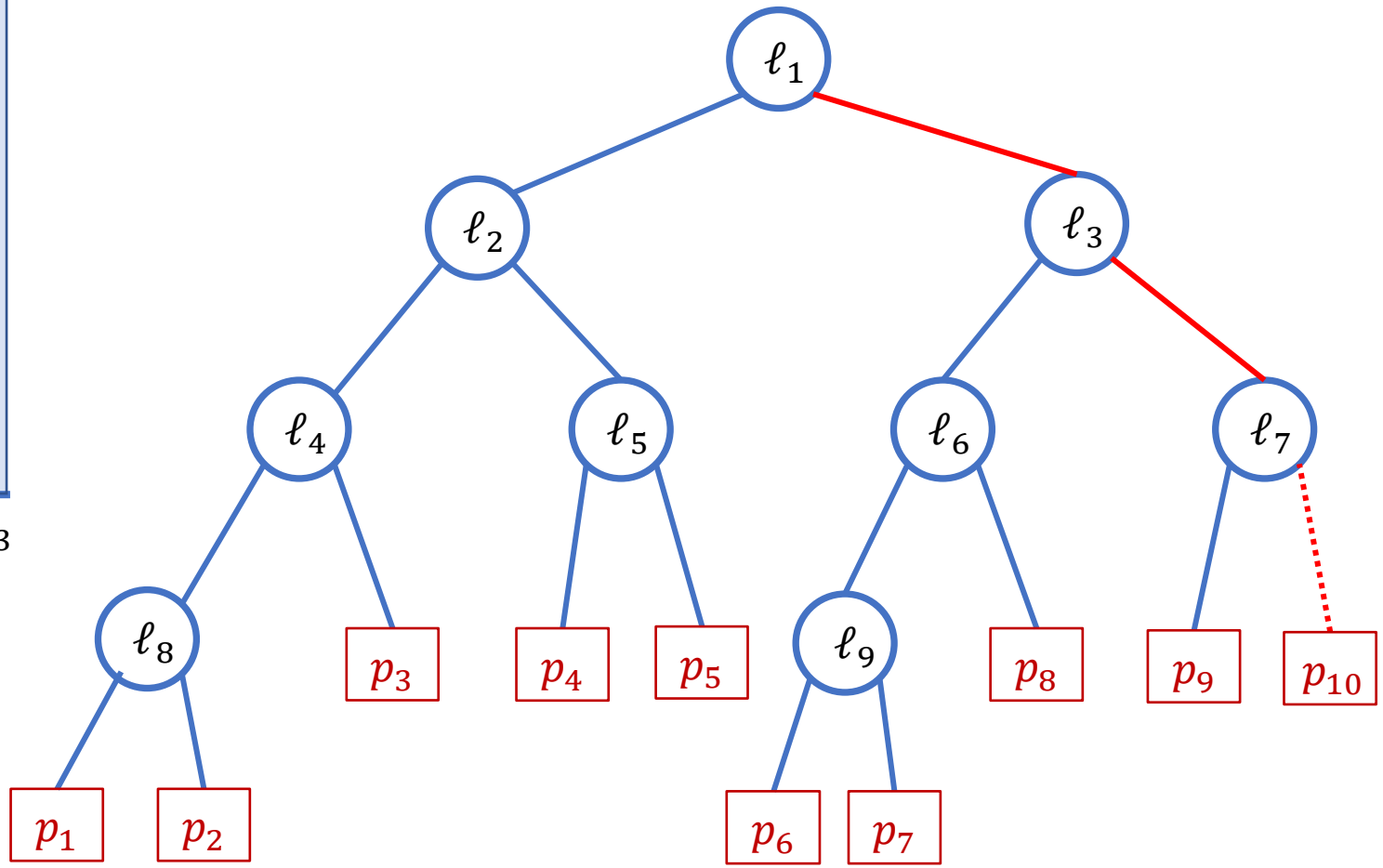

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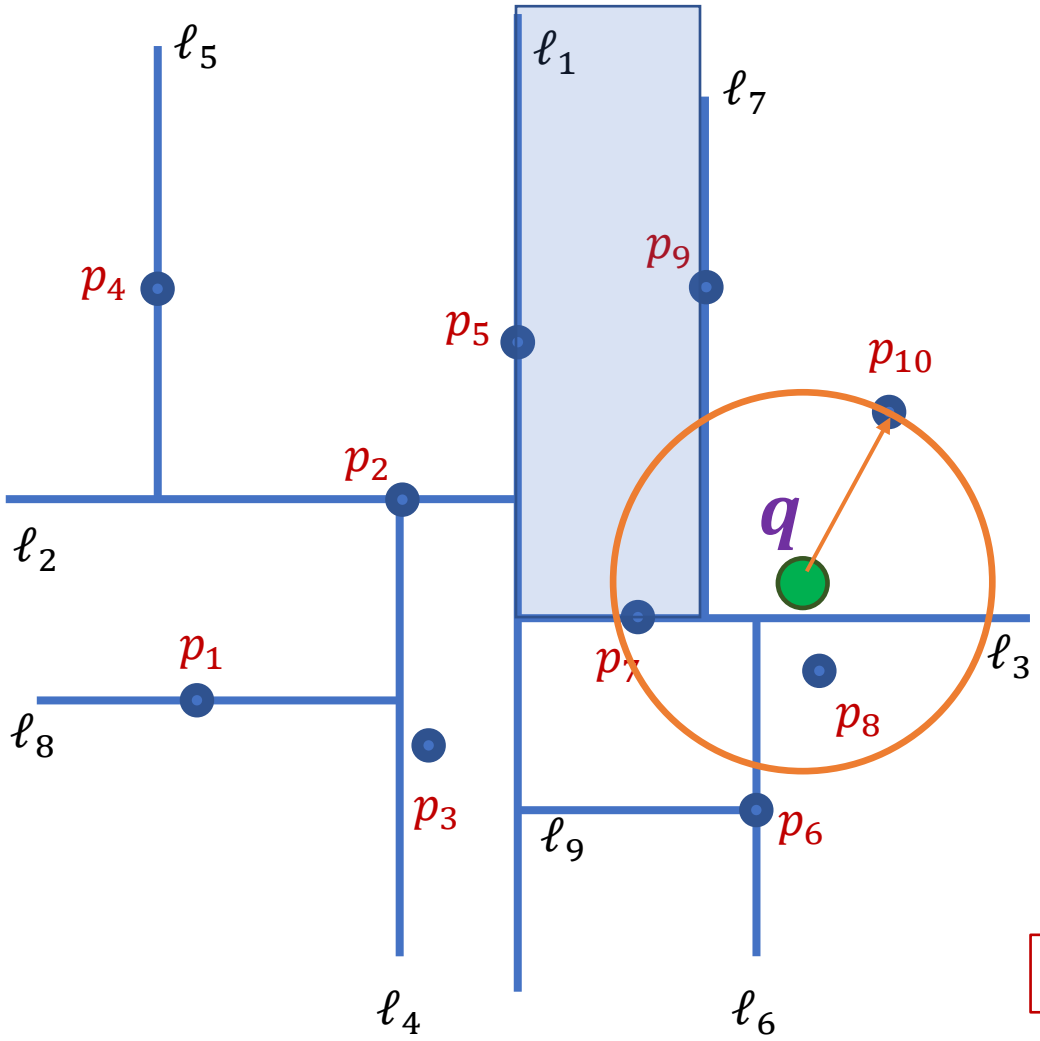

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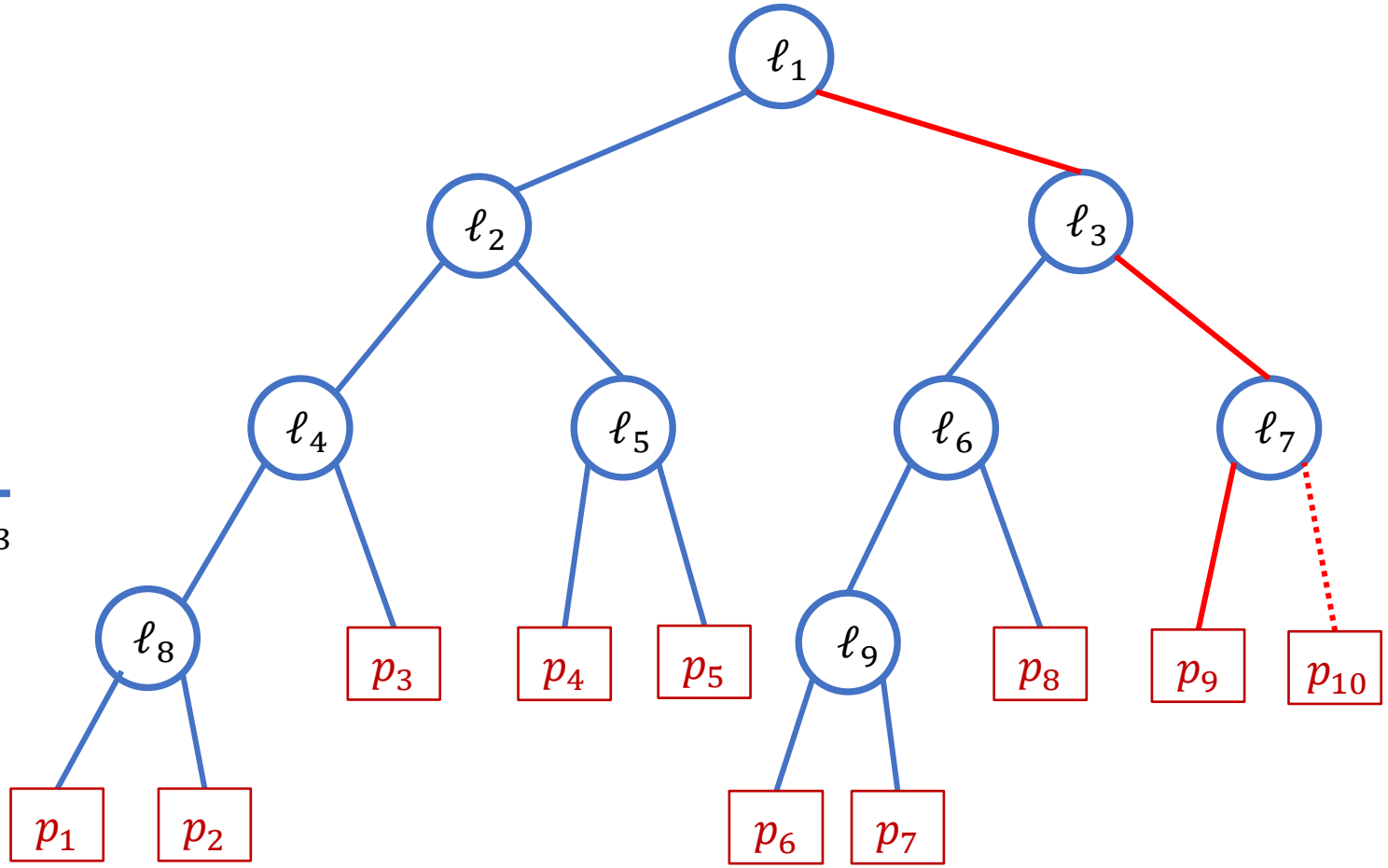

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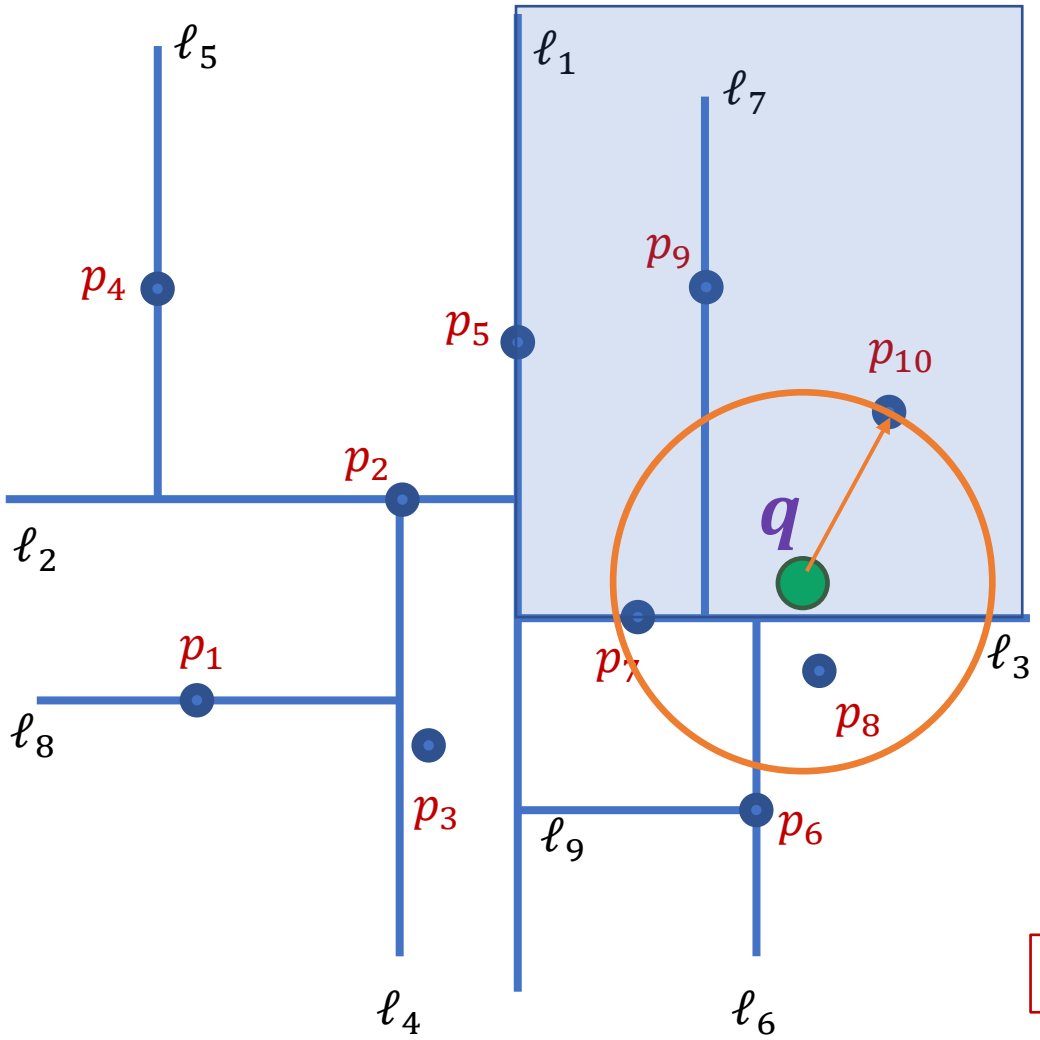

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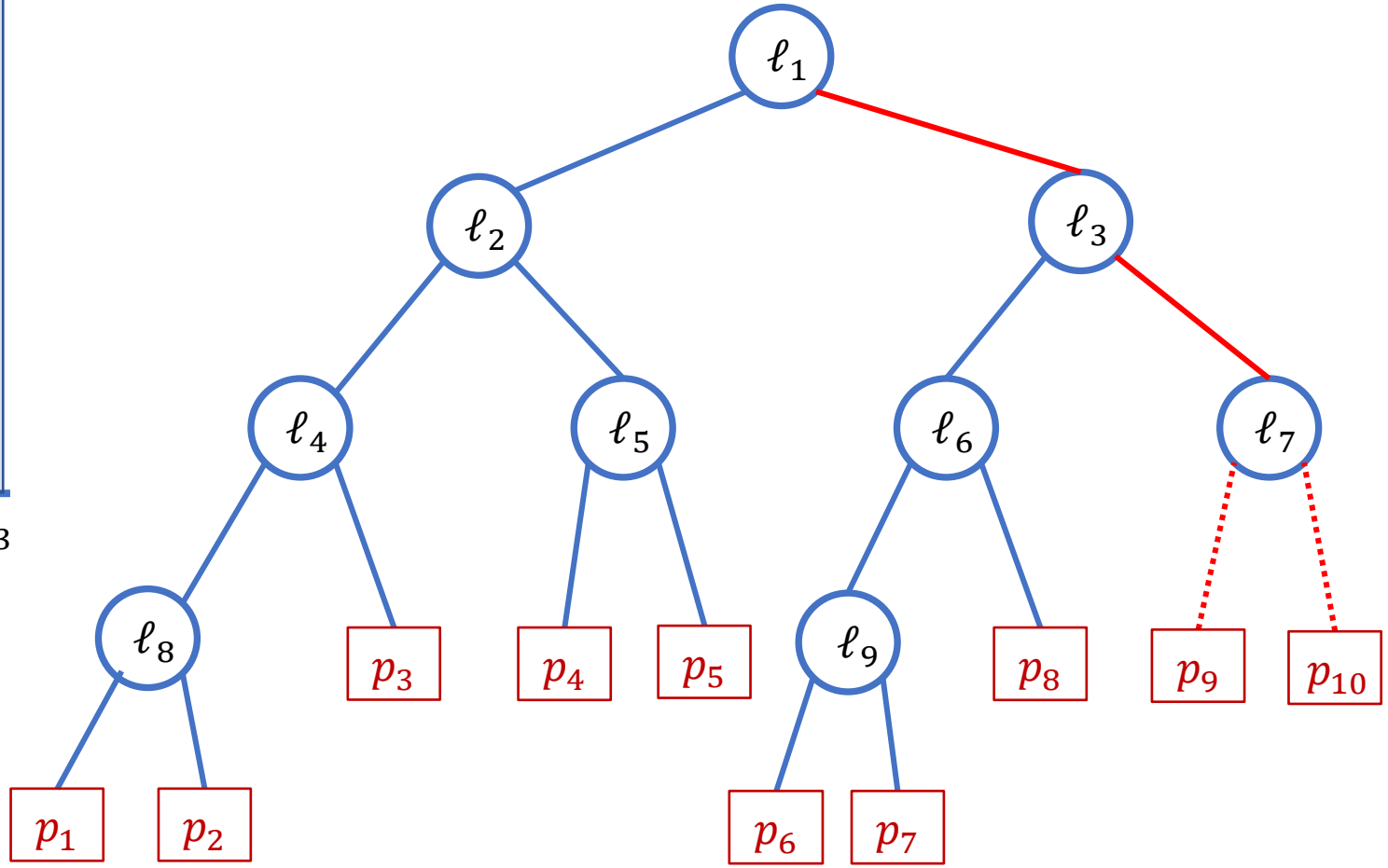

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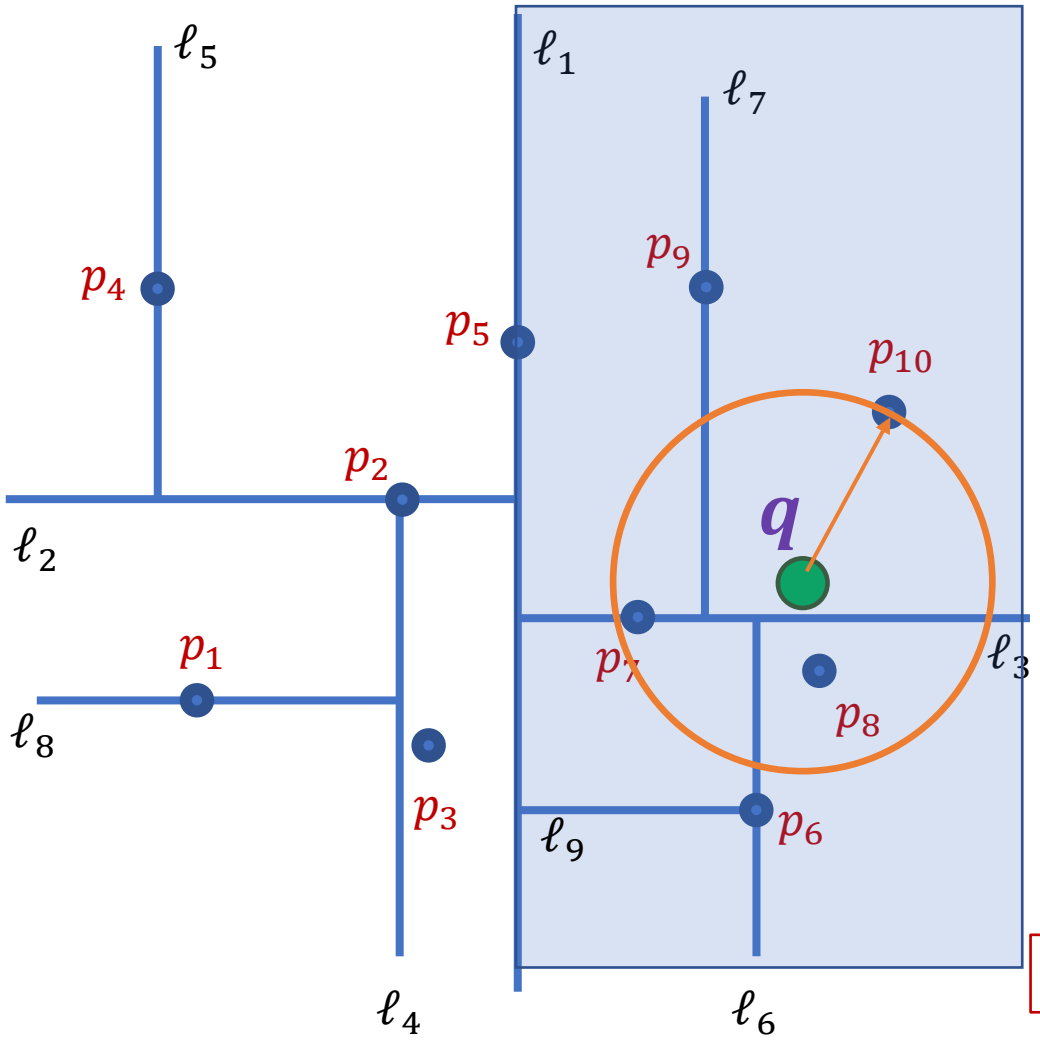

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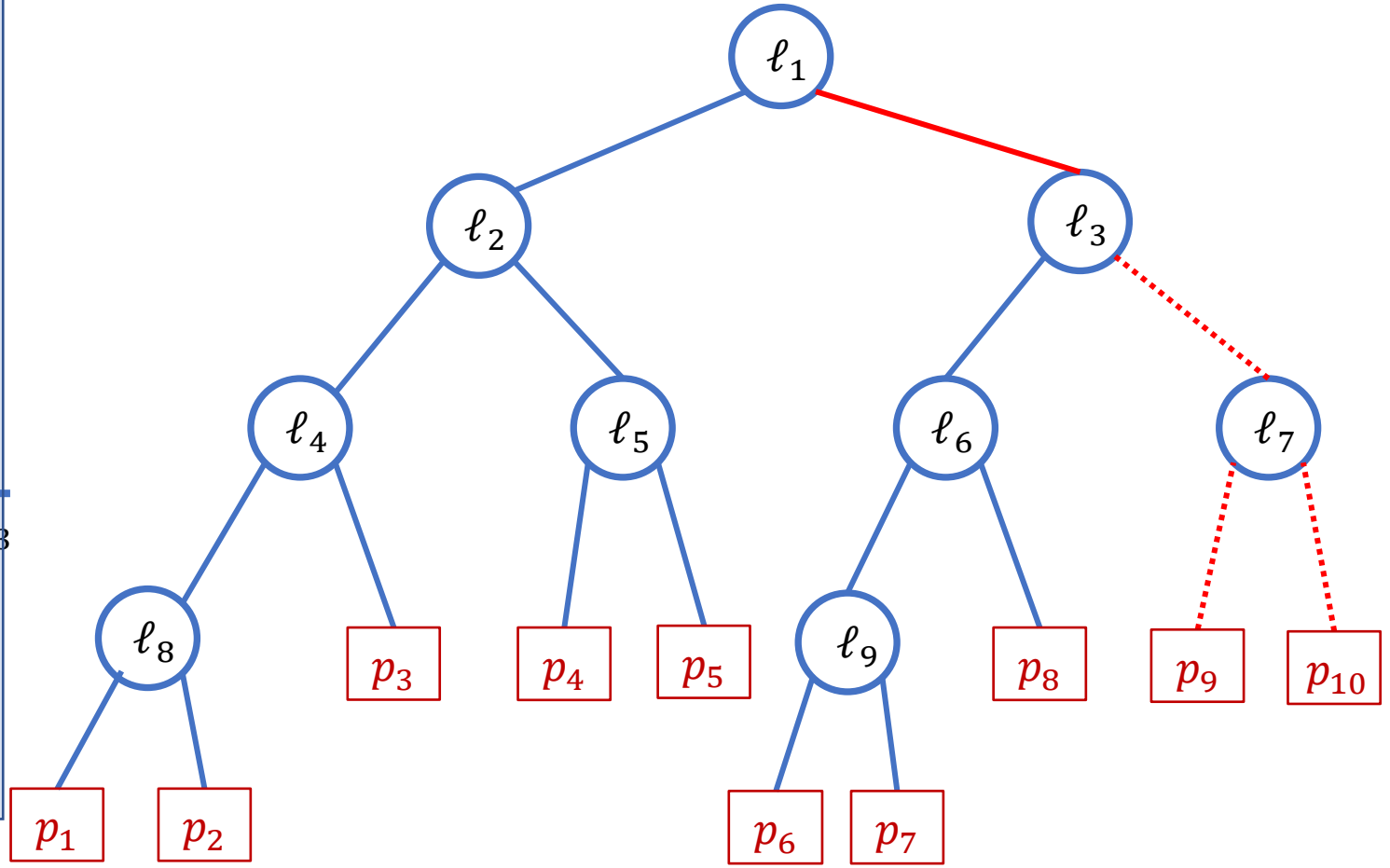

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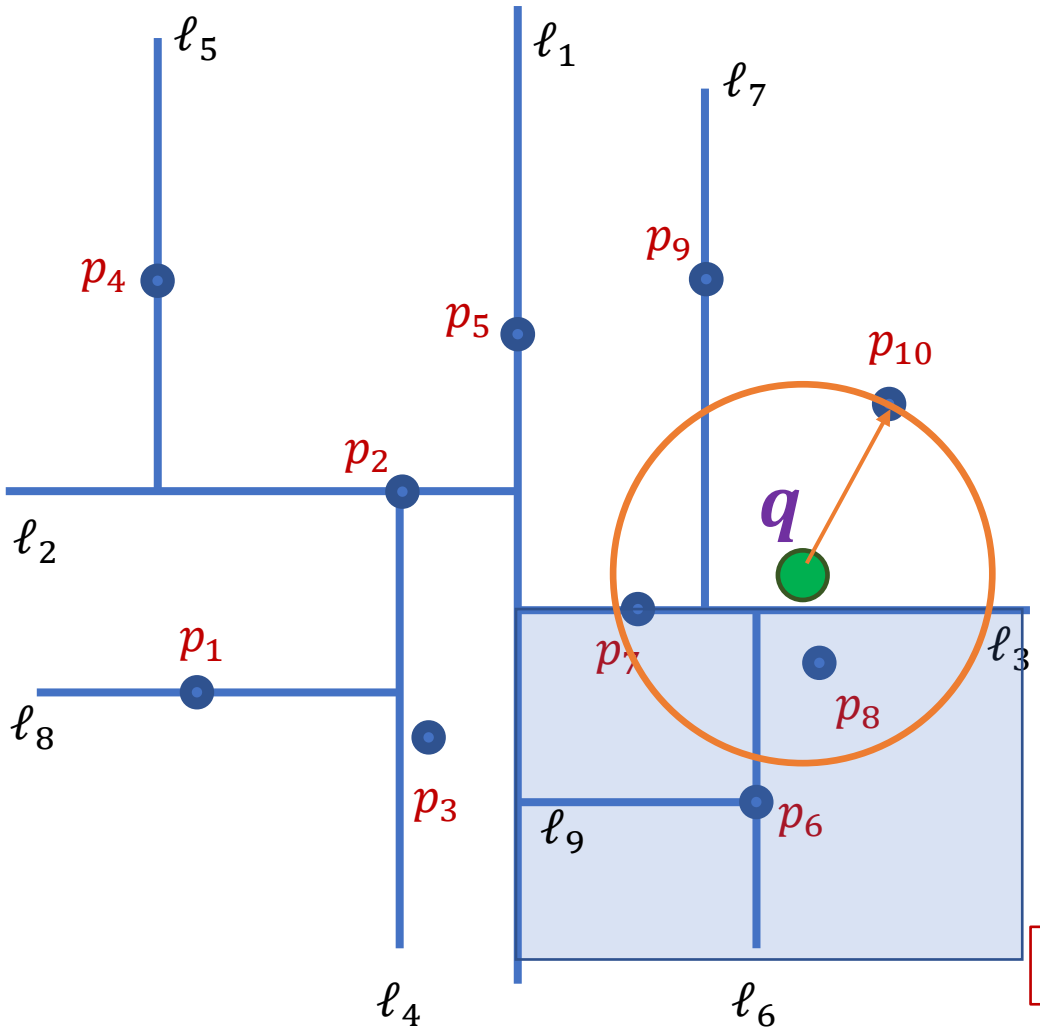

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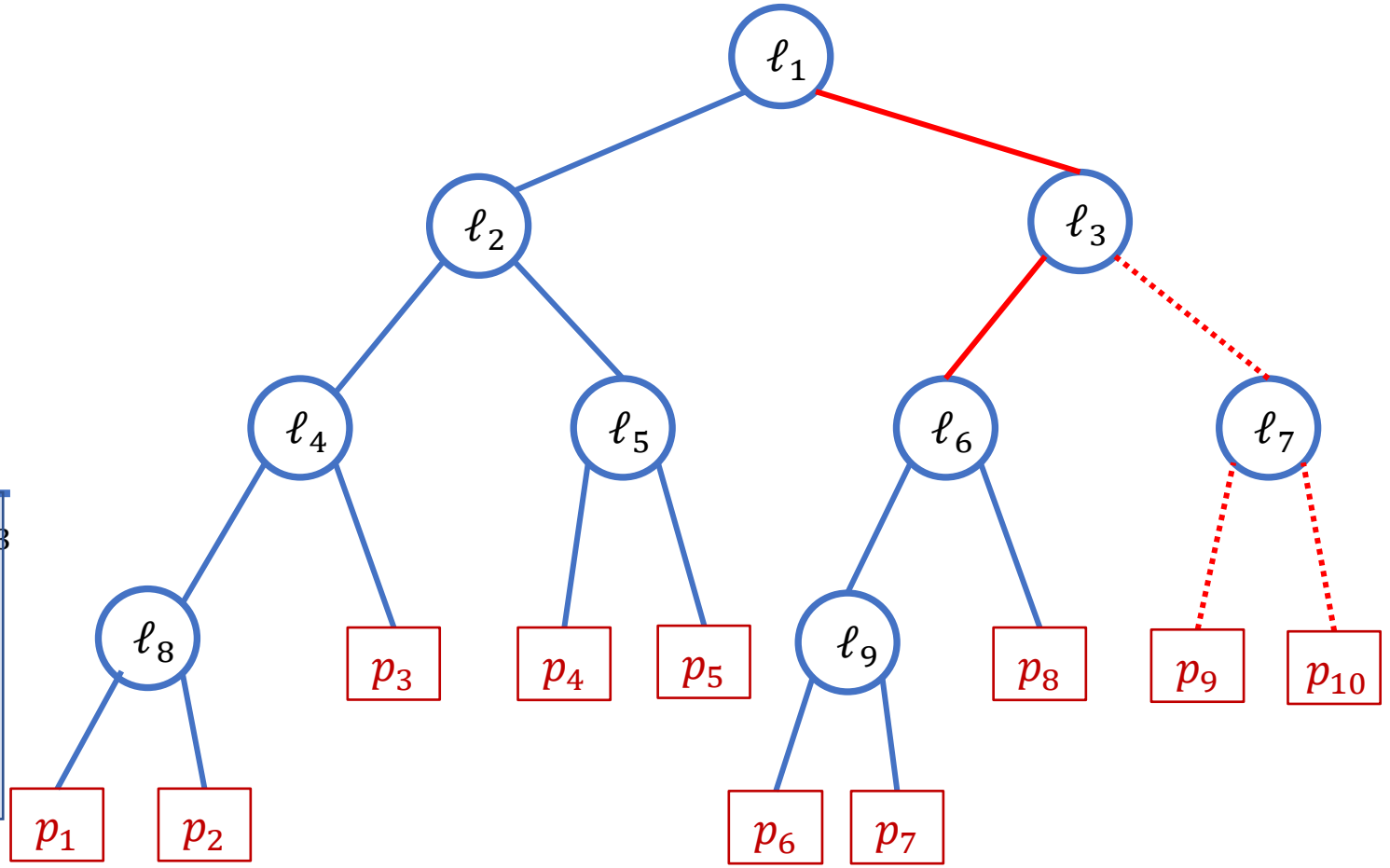

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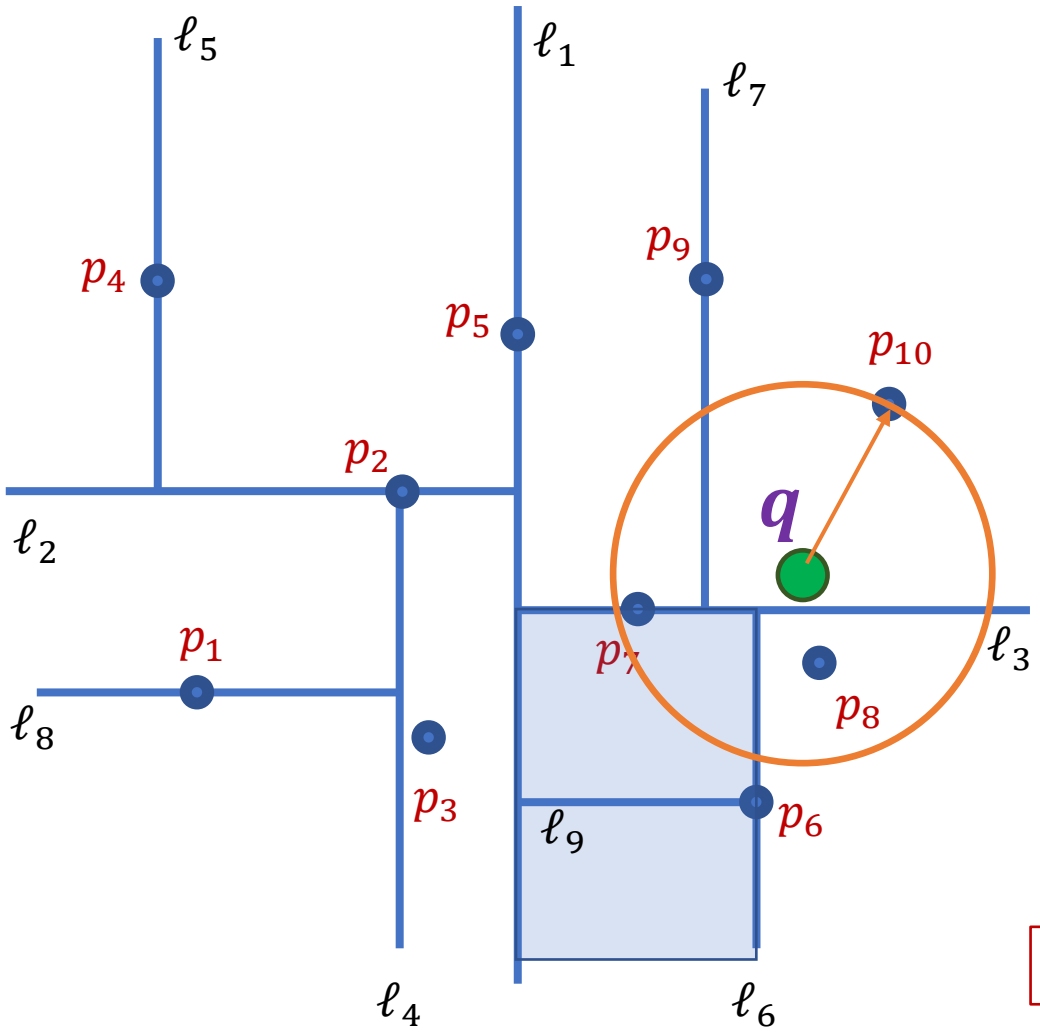

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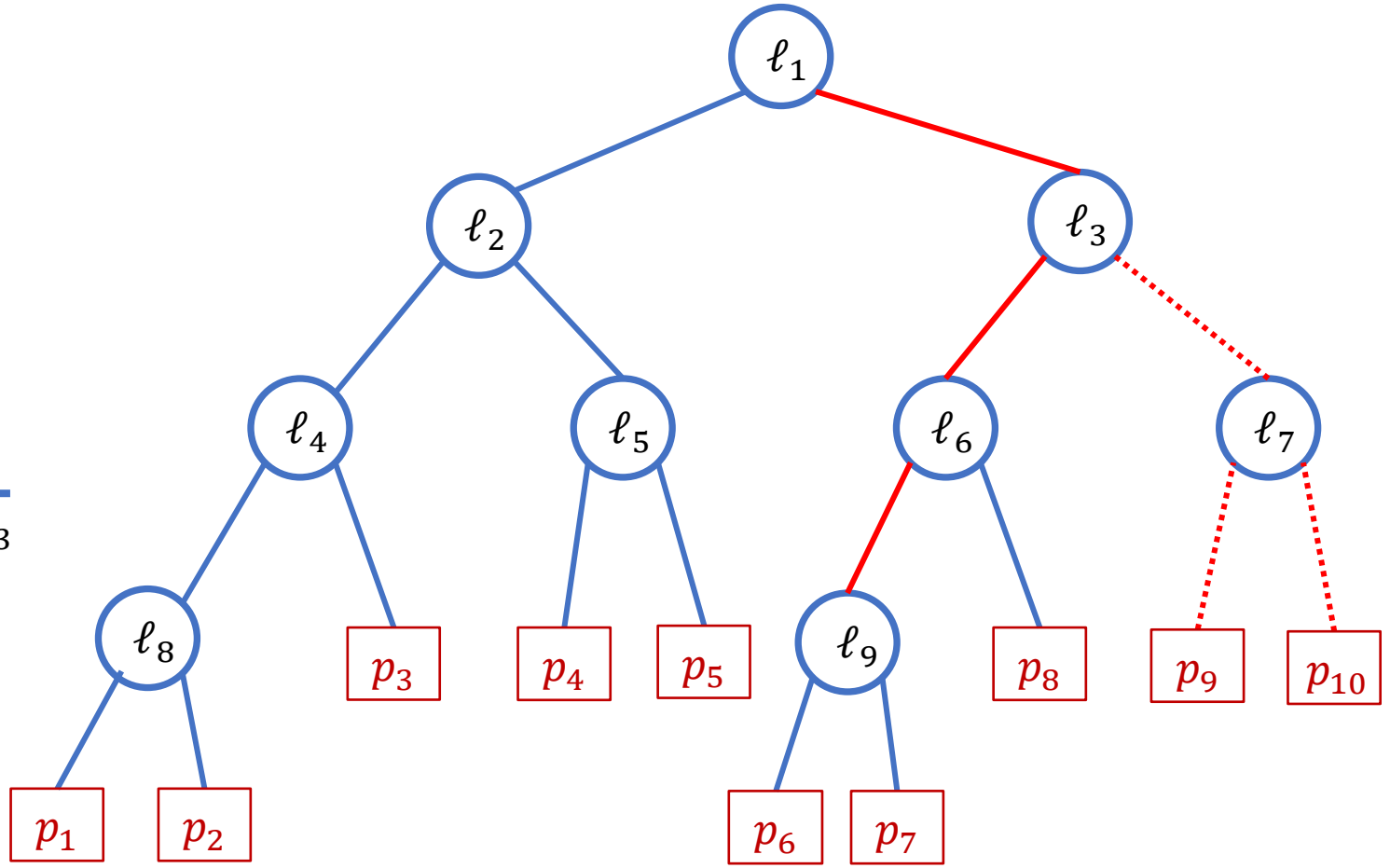

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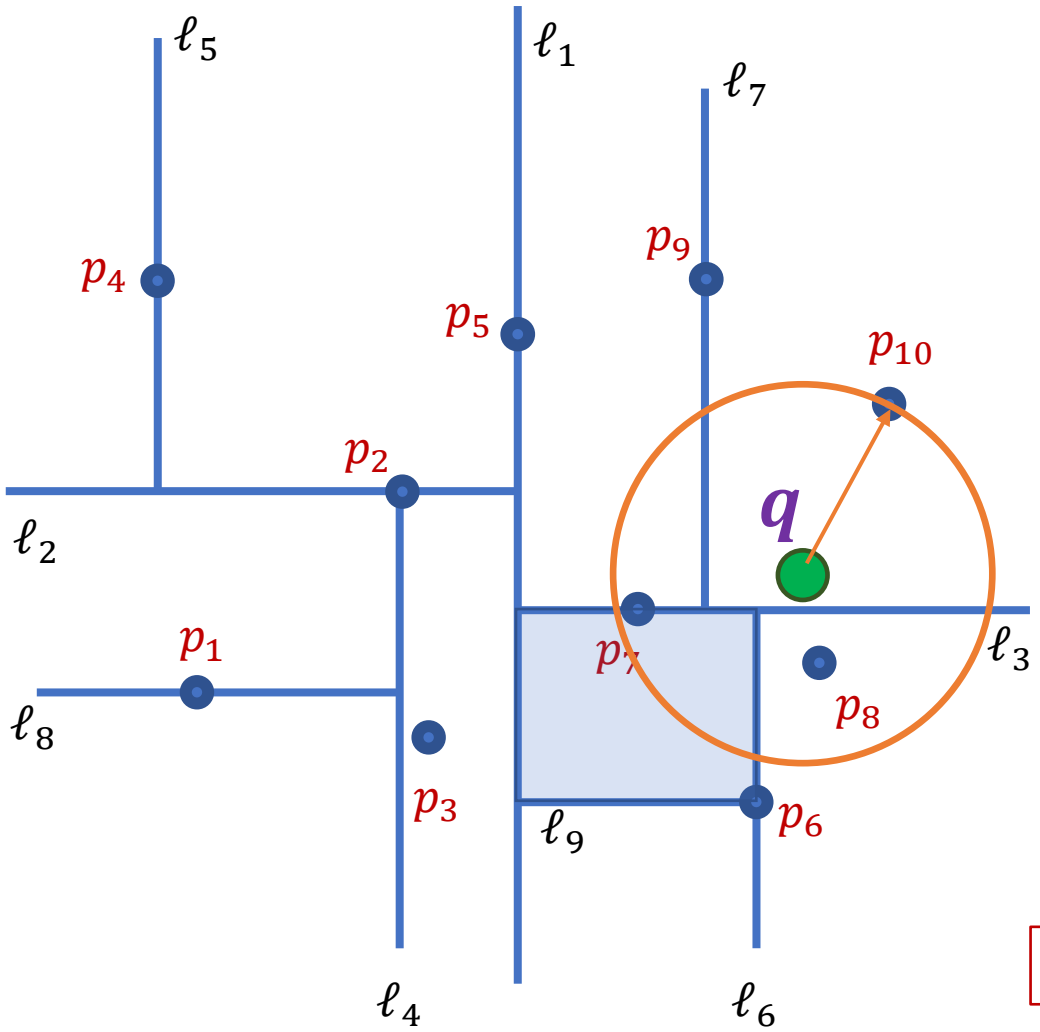

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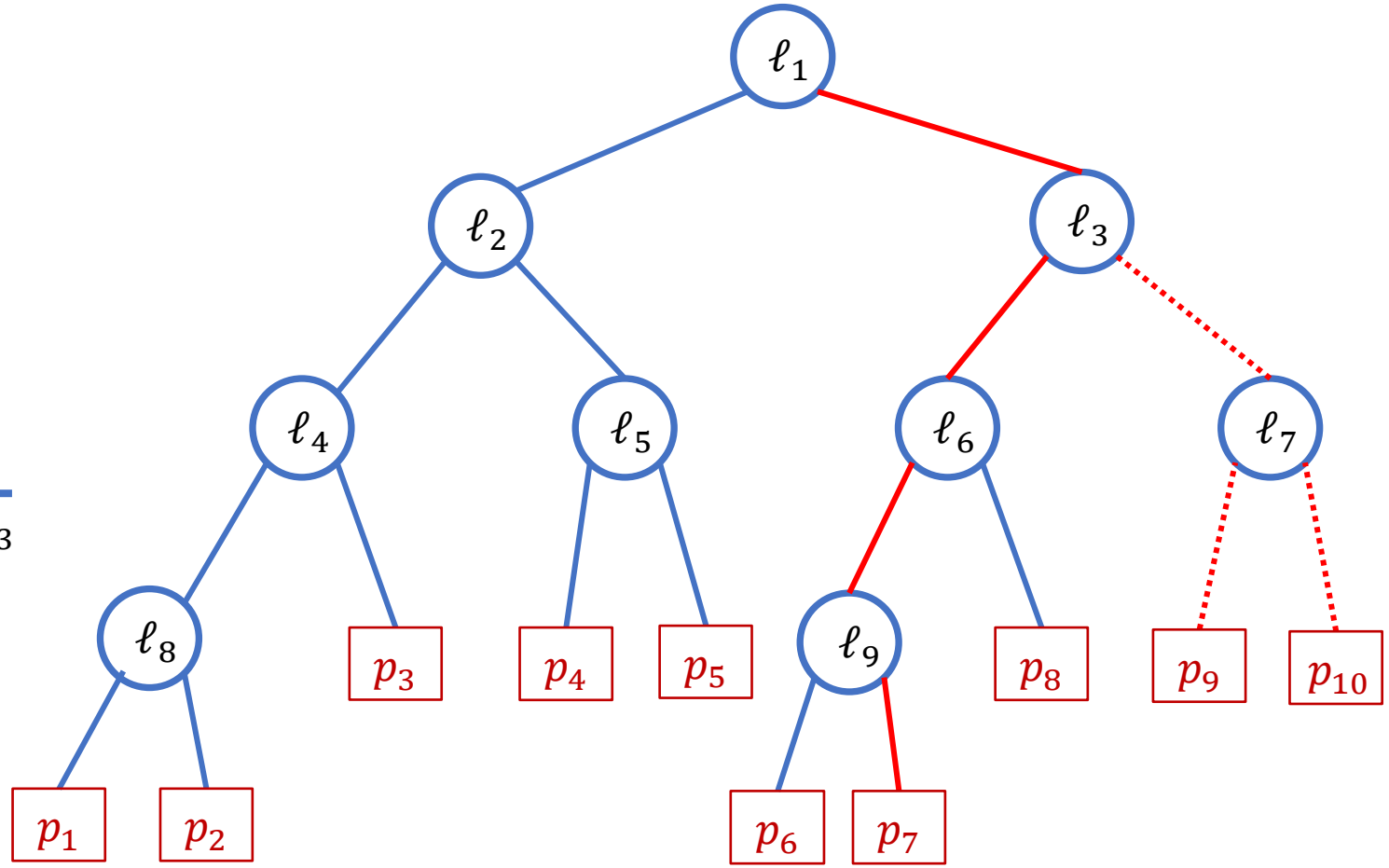
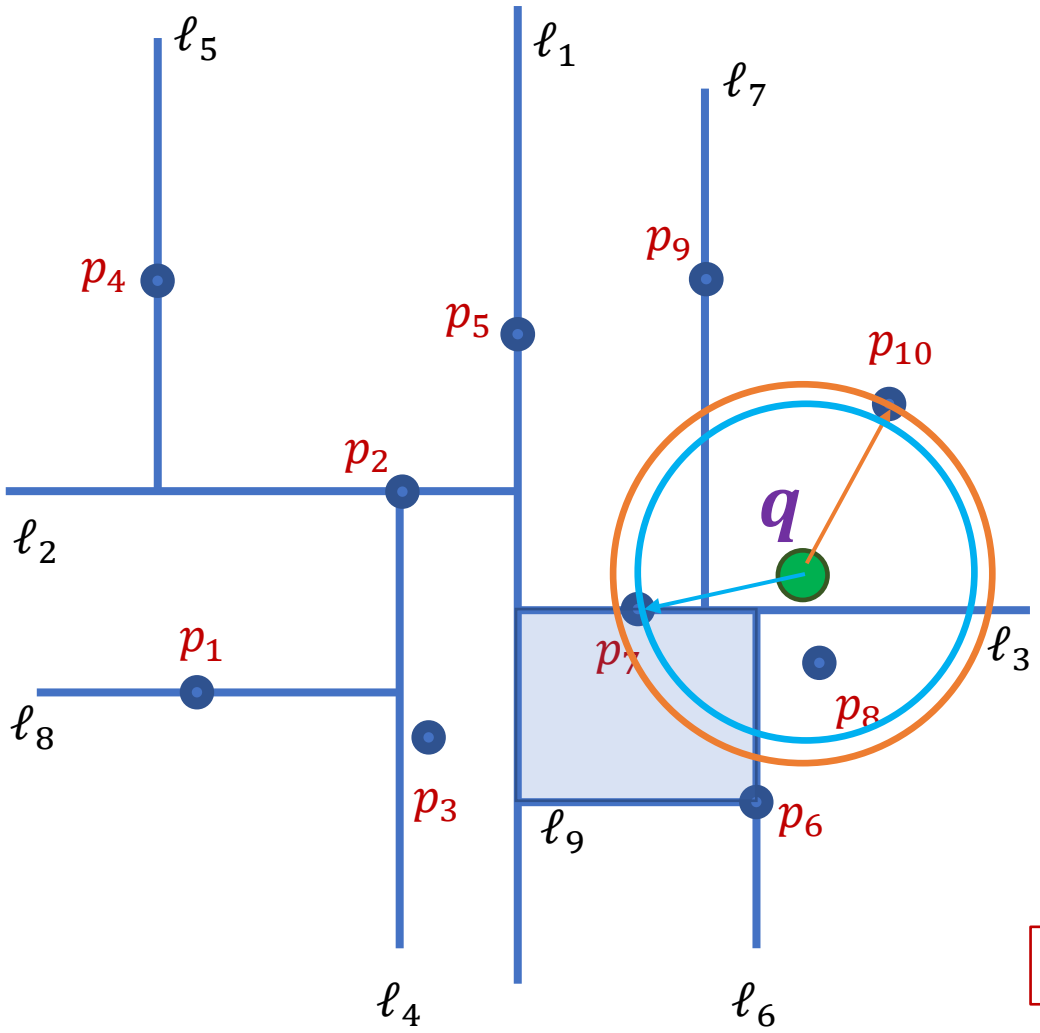
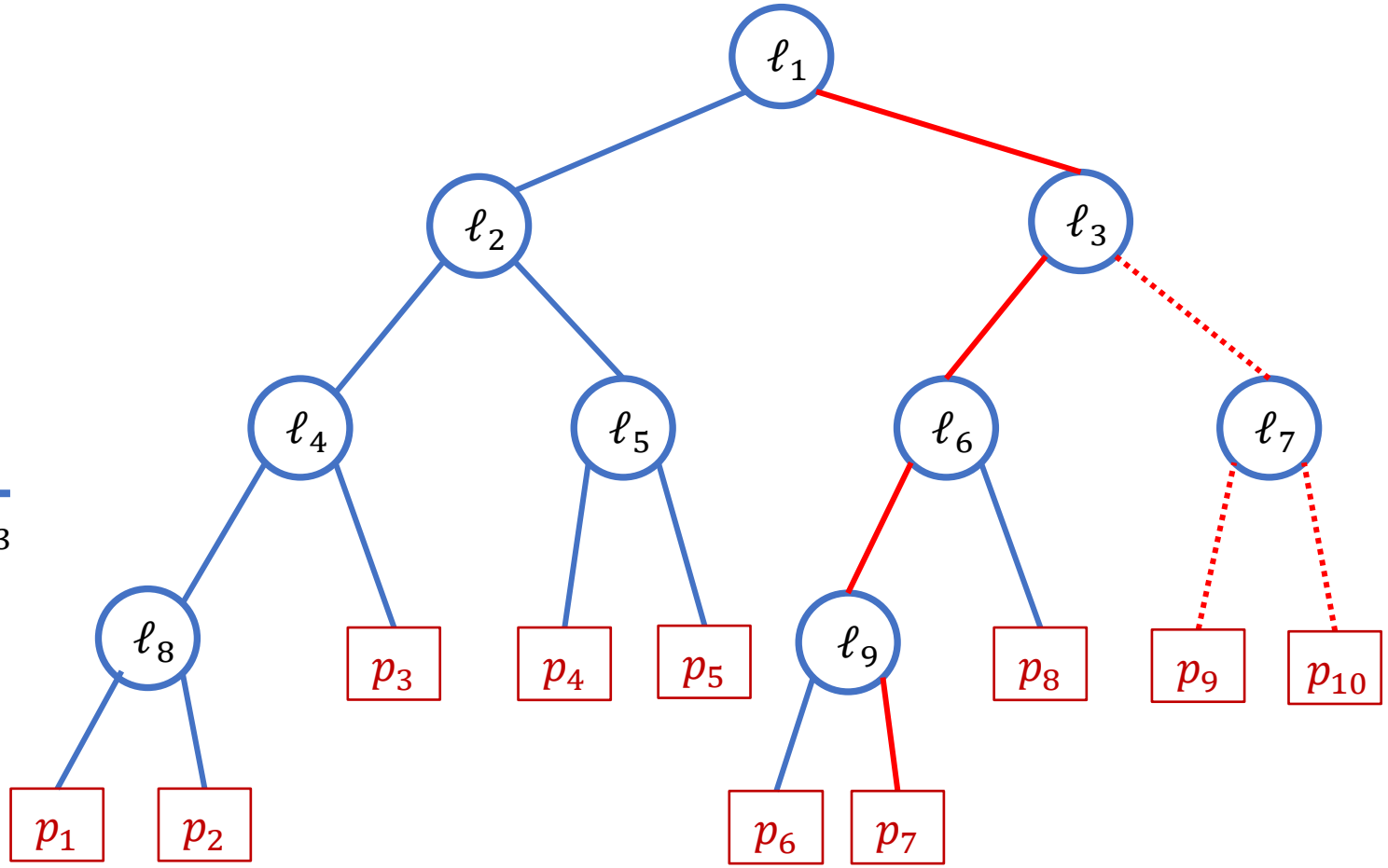


Updating the candidate to p_7

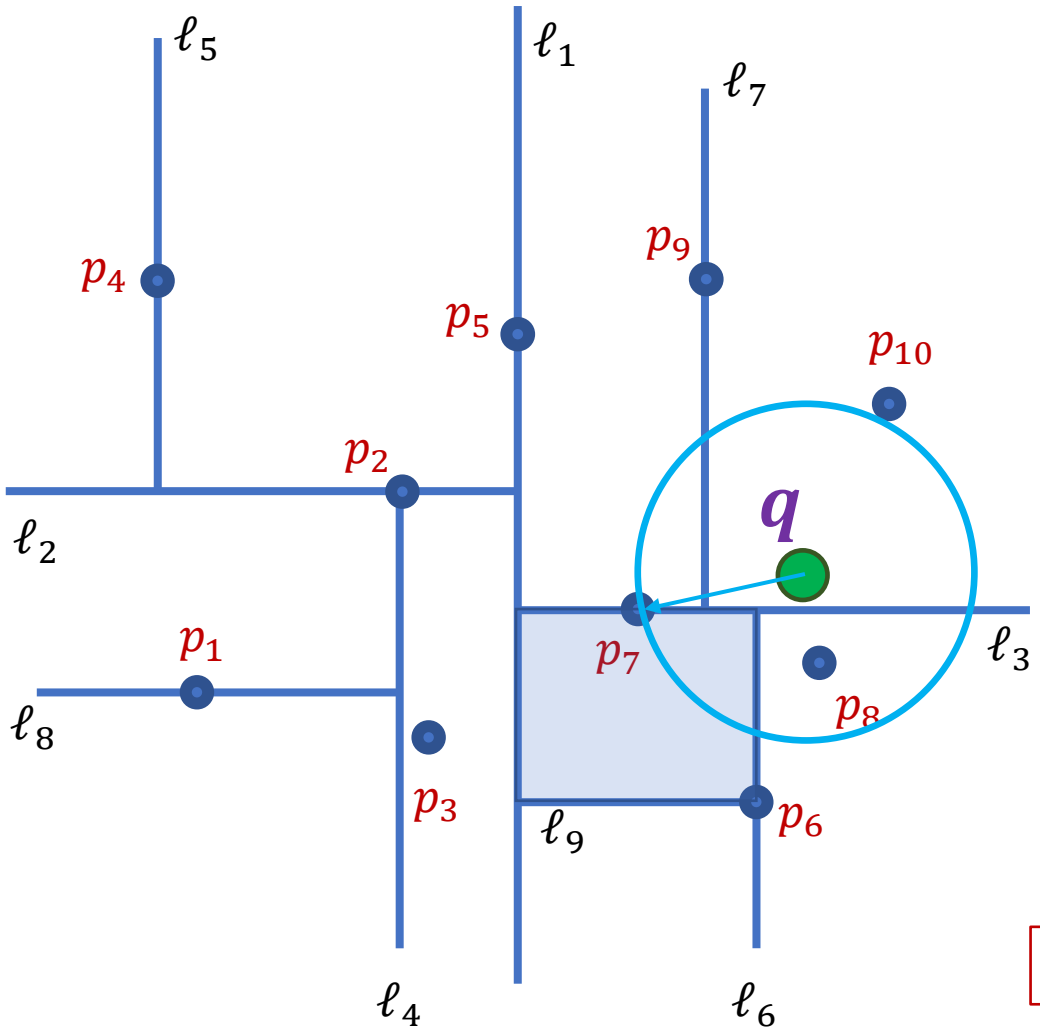
Current candidate: p_7

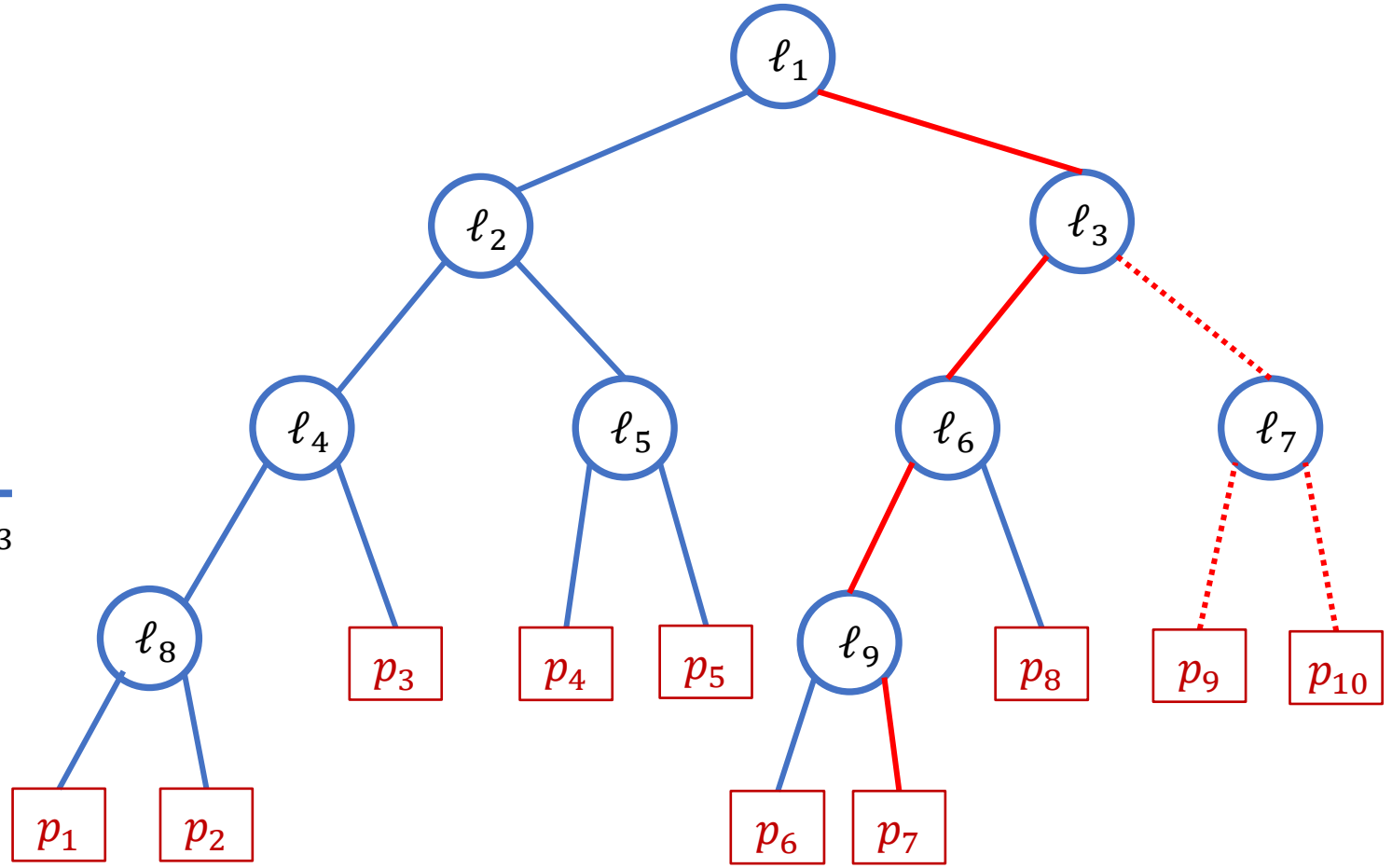




Updating the candidate to p_7

Current candidate: p_7





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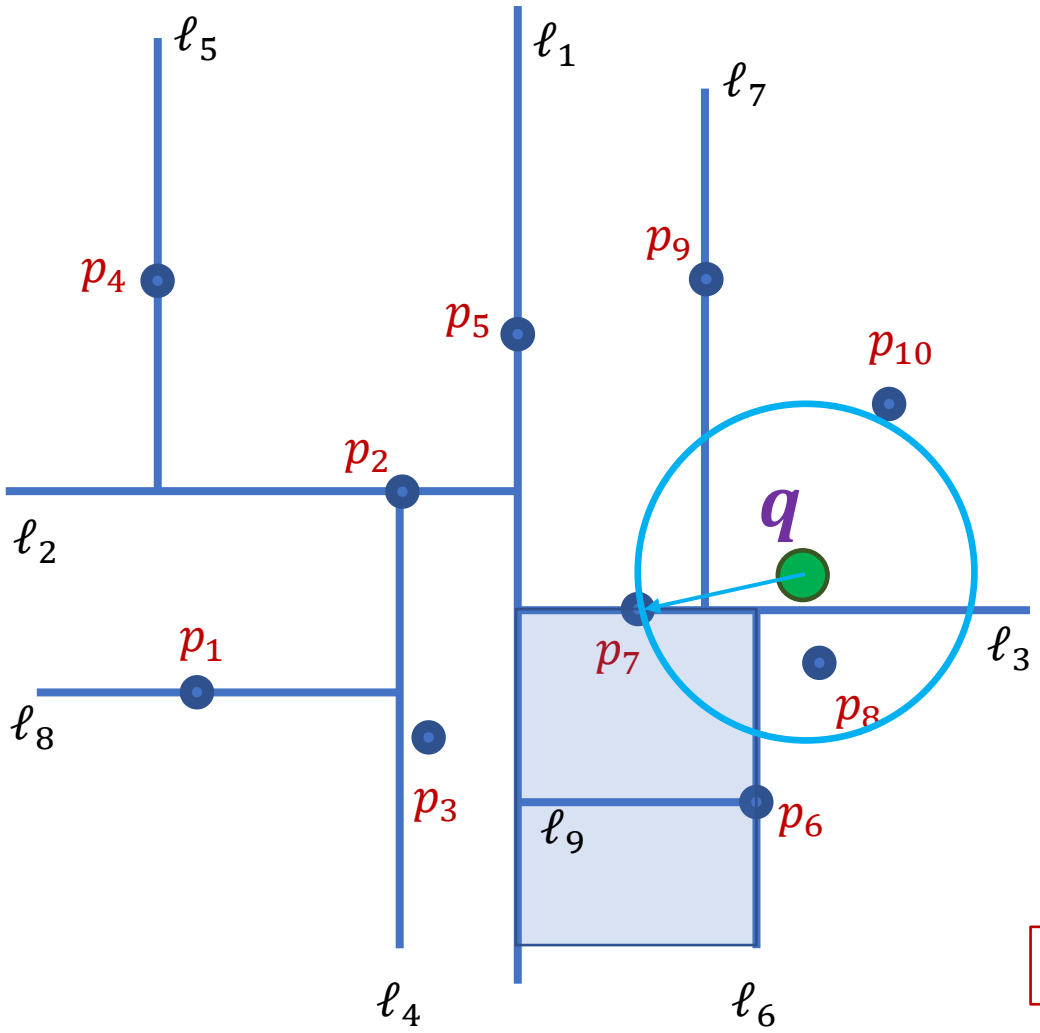

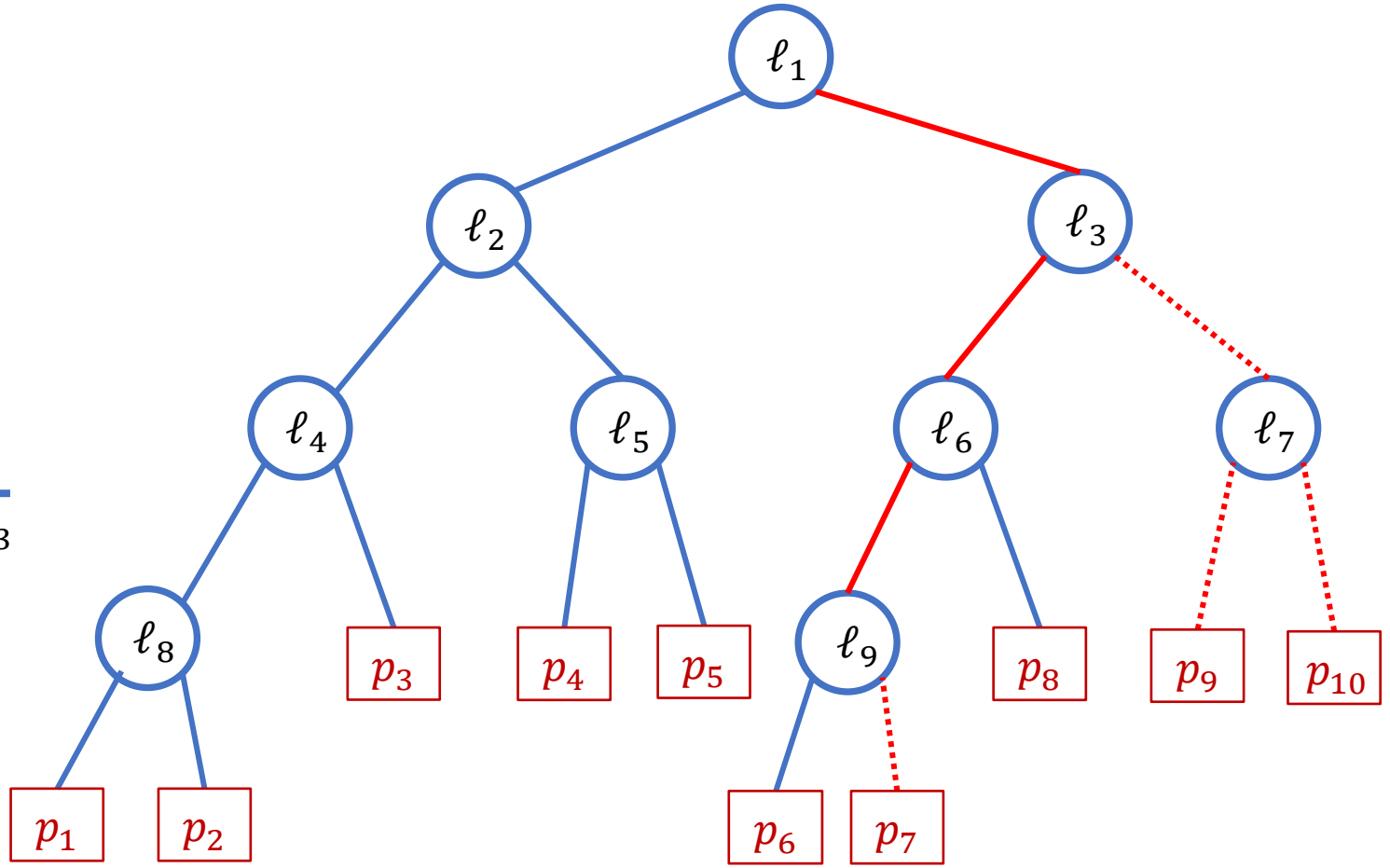

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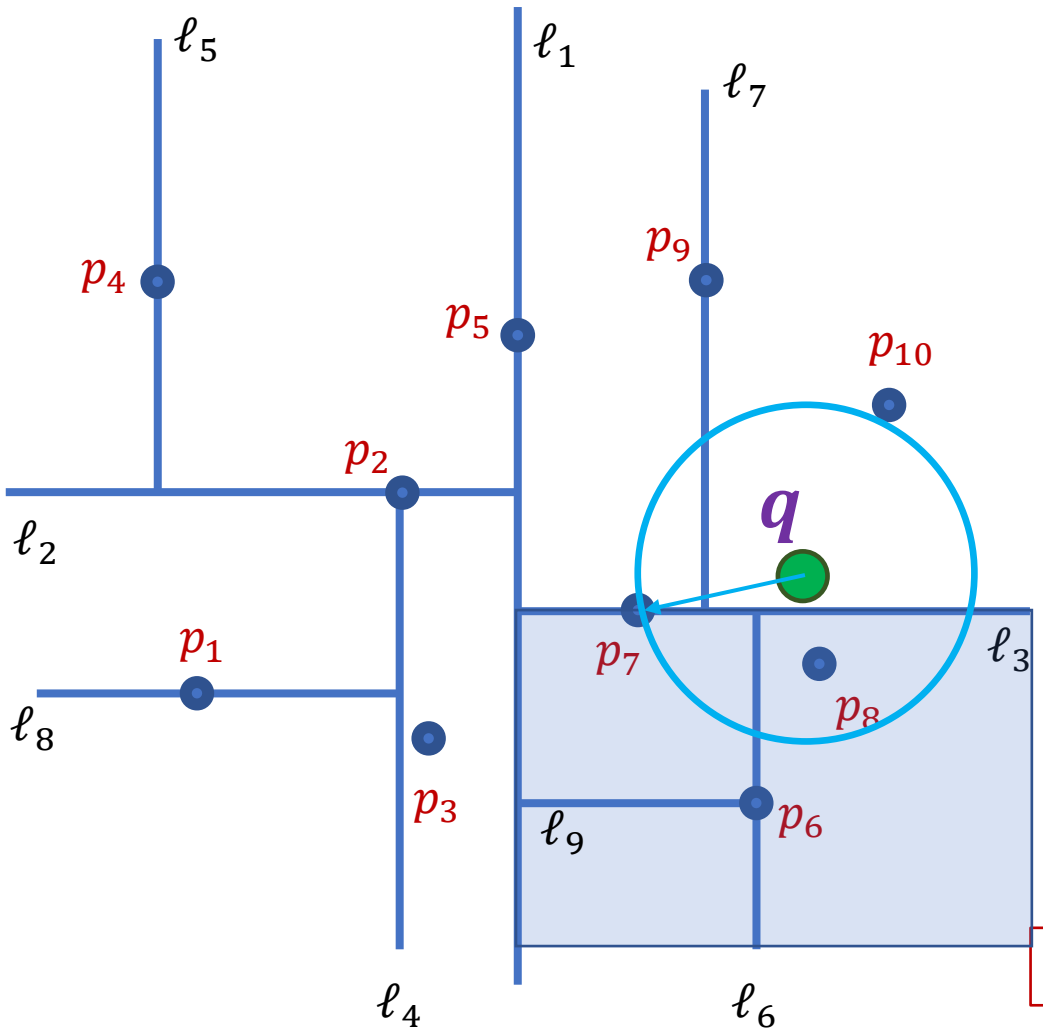

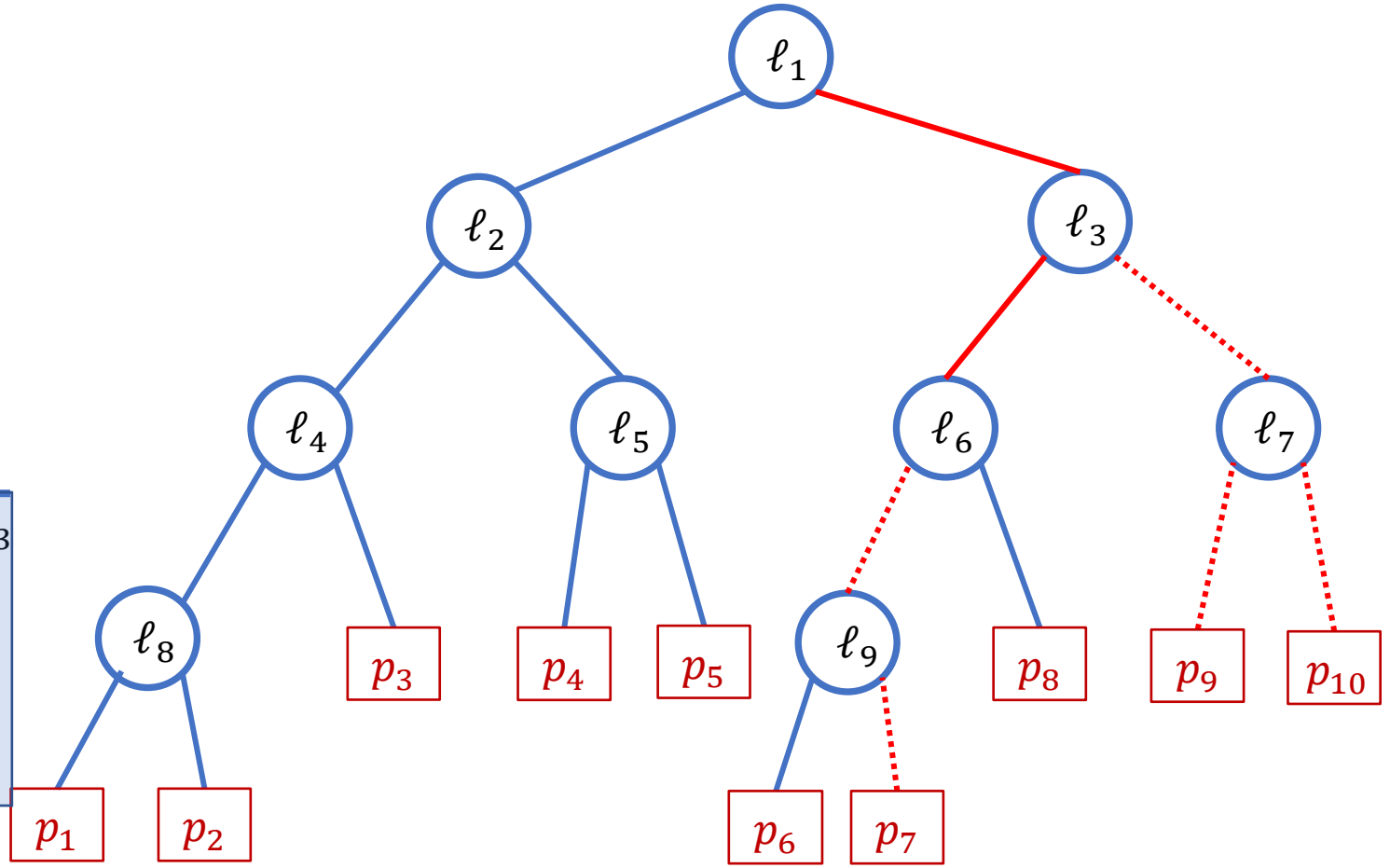

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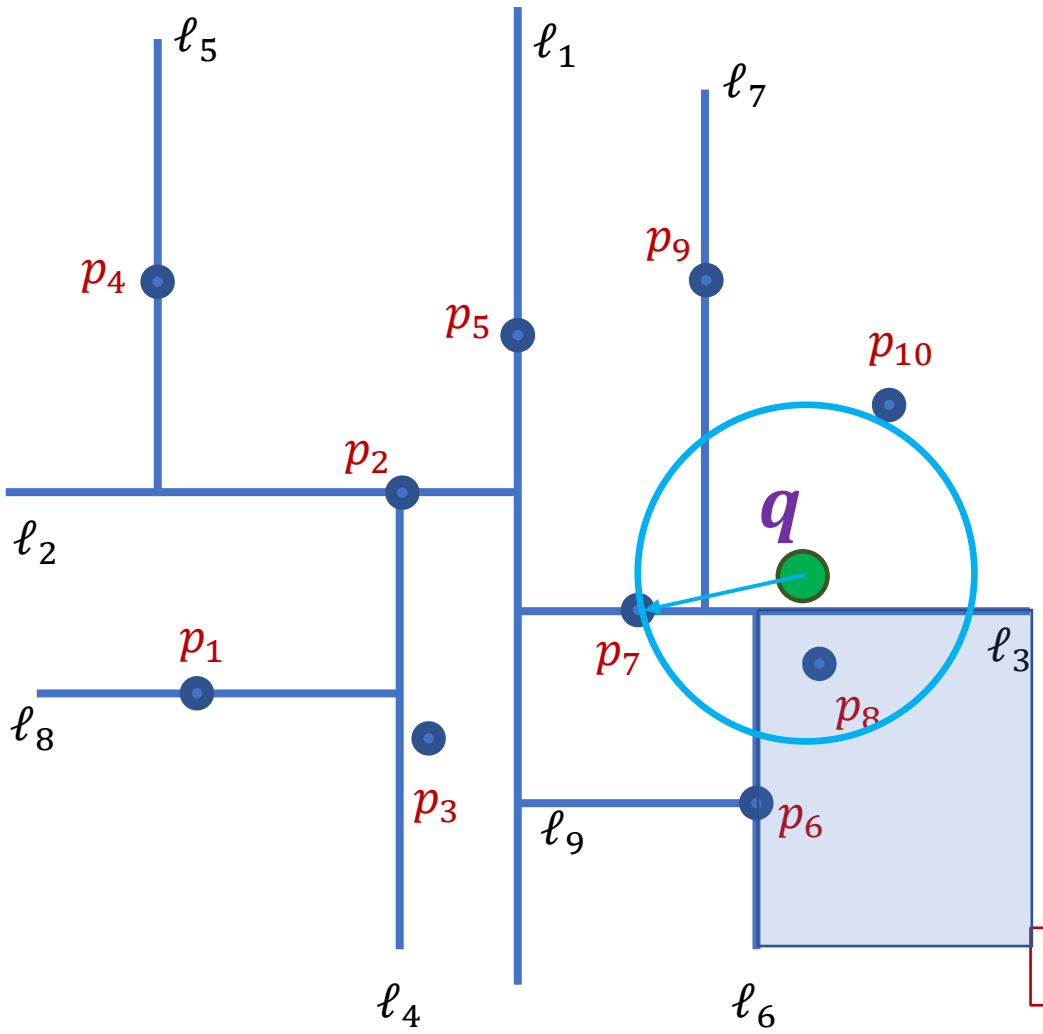
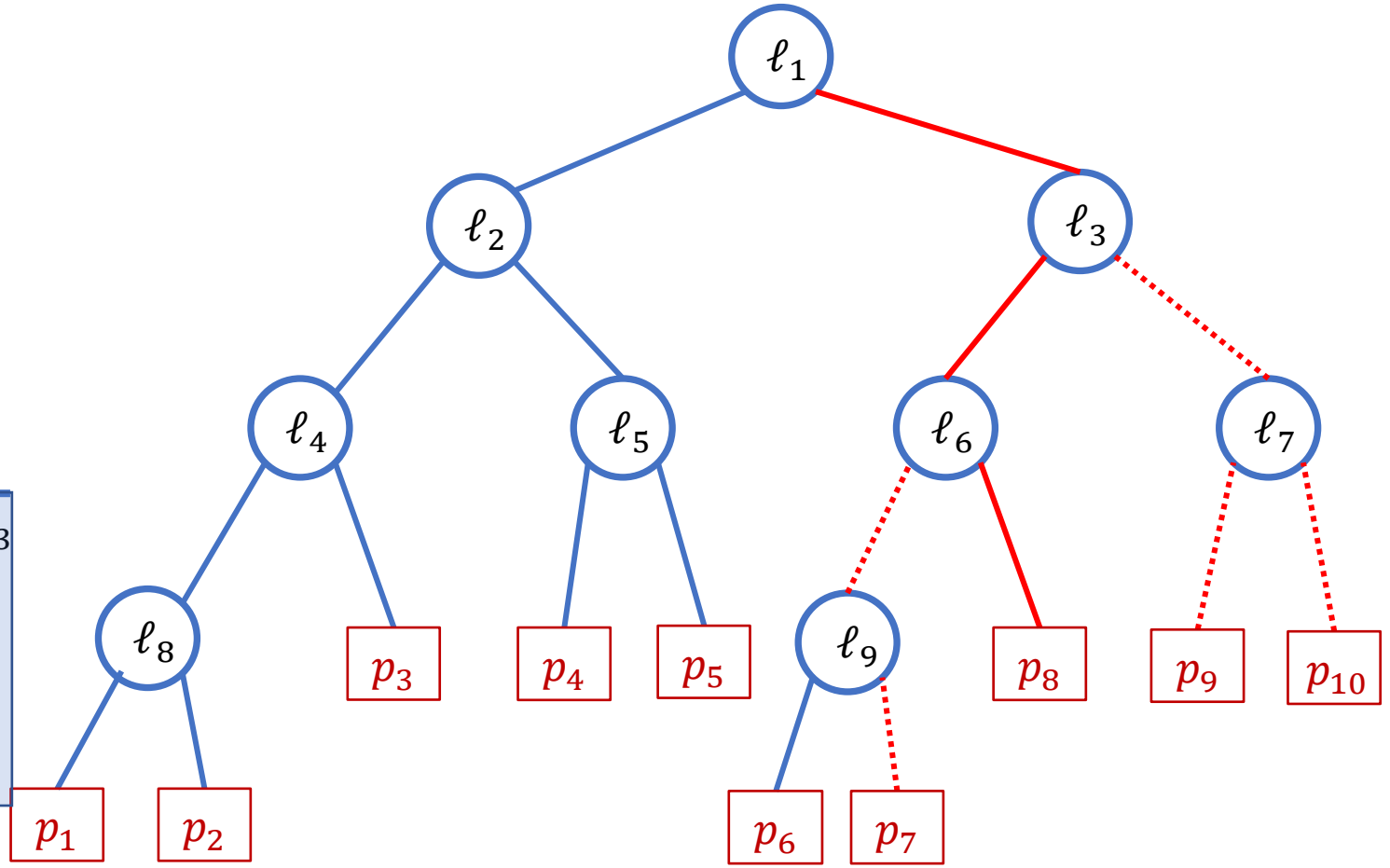
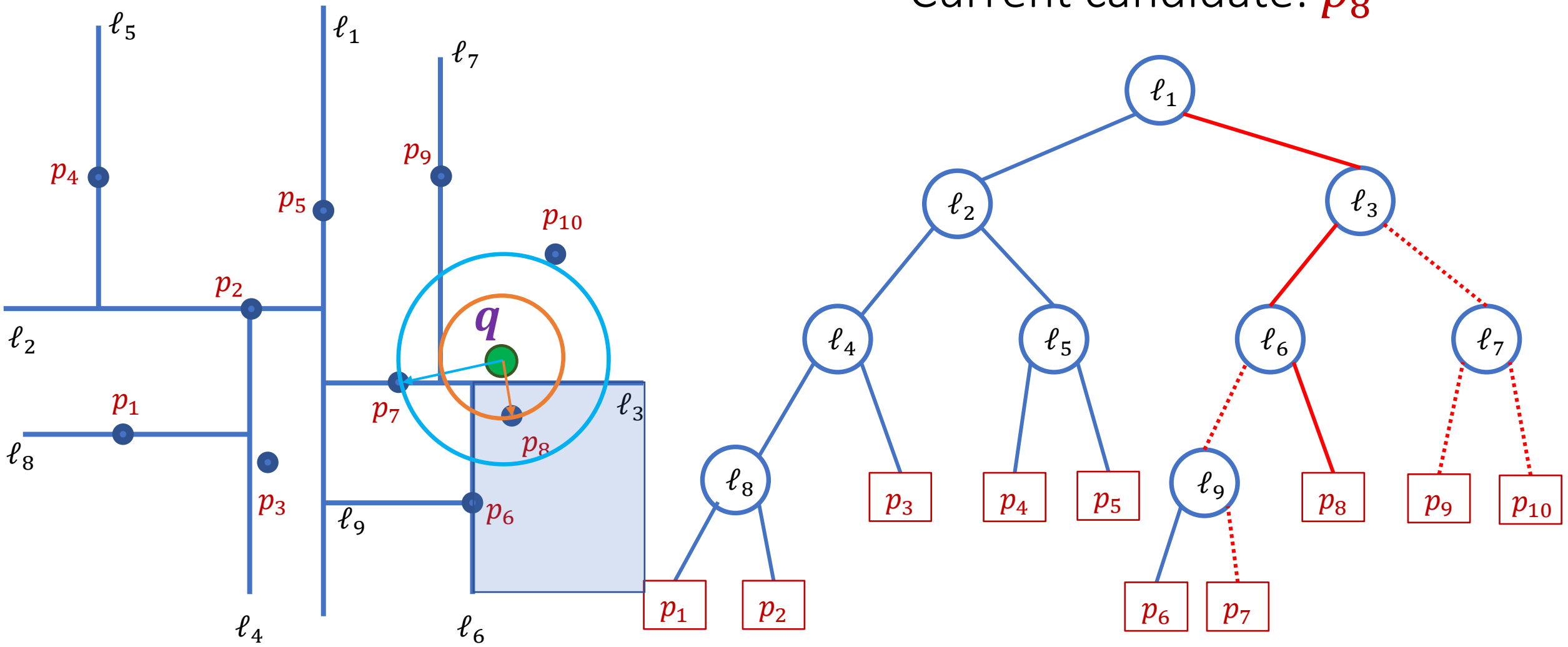




Updating the candidate to p_8

Current candidate: p_8



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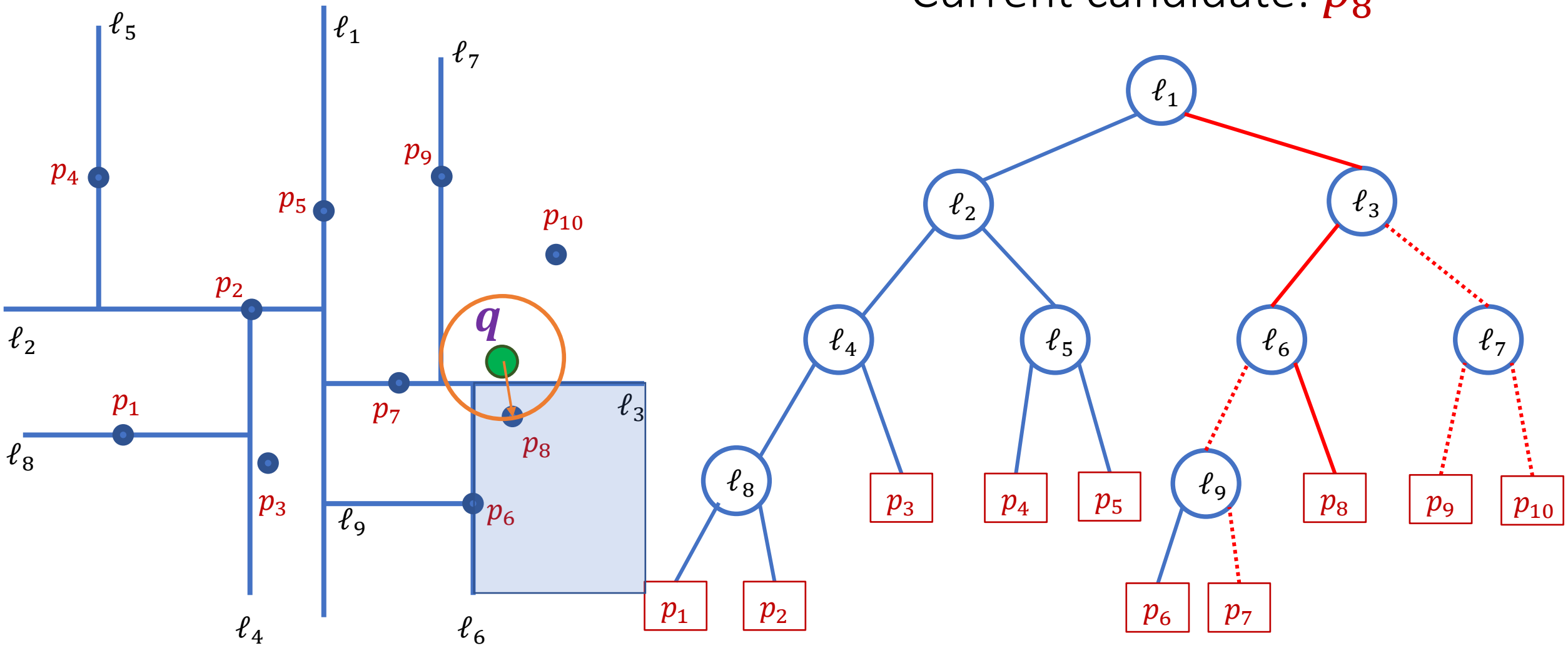
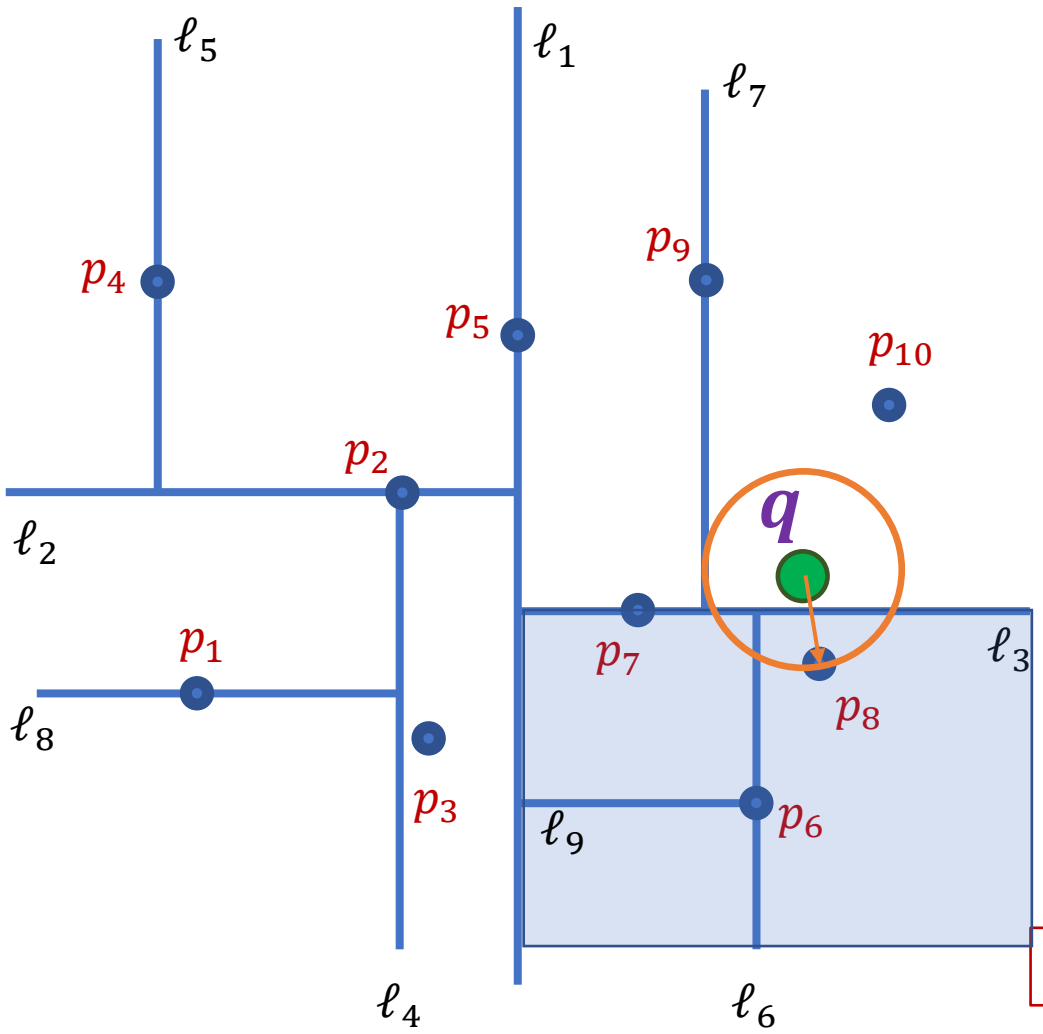
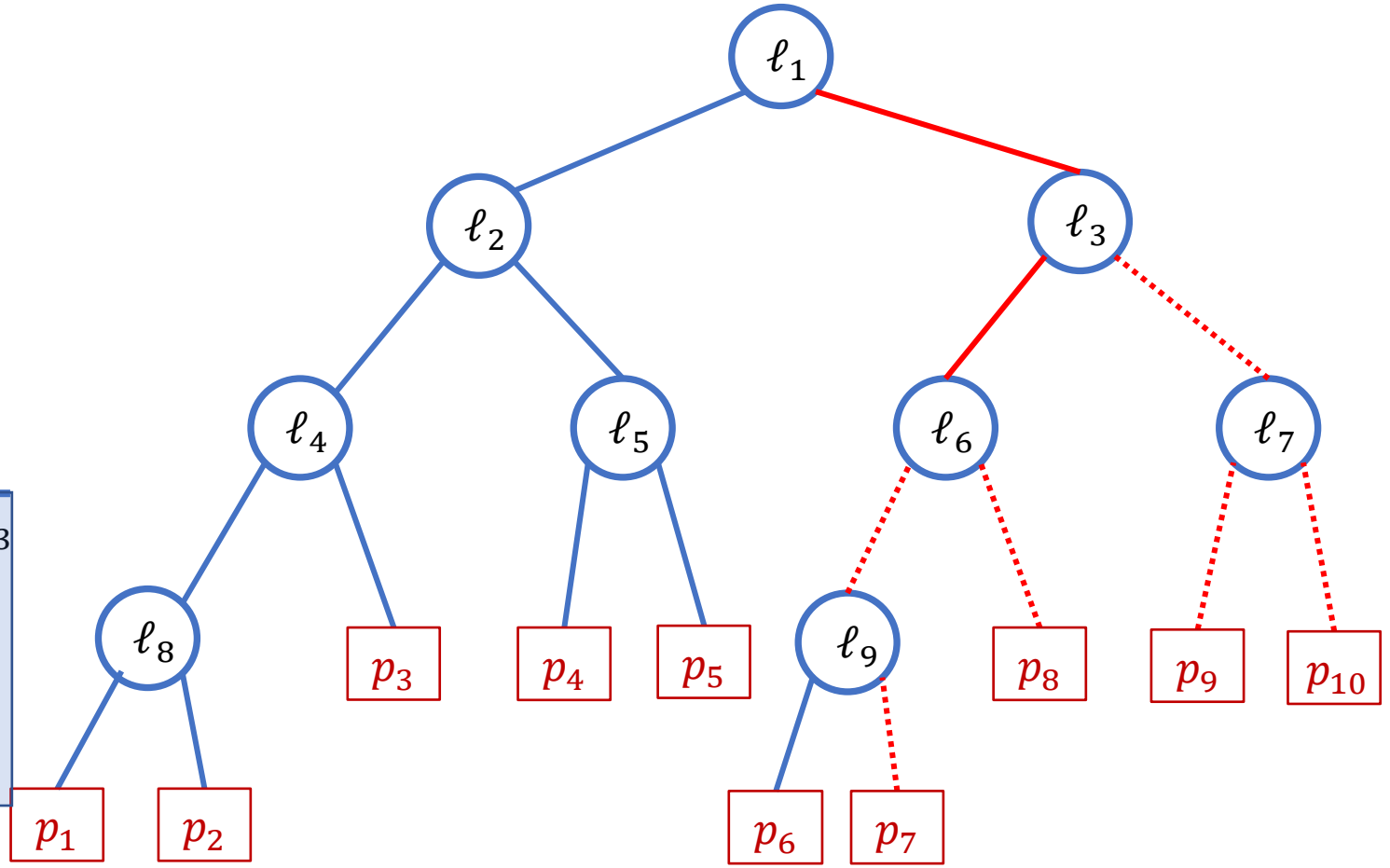
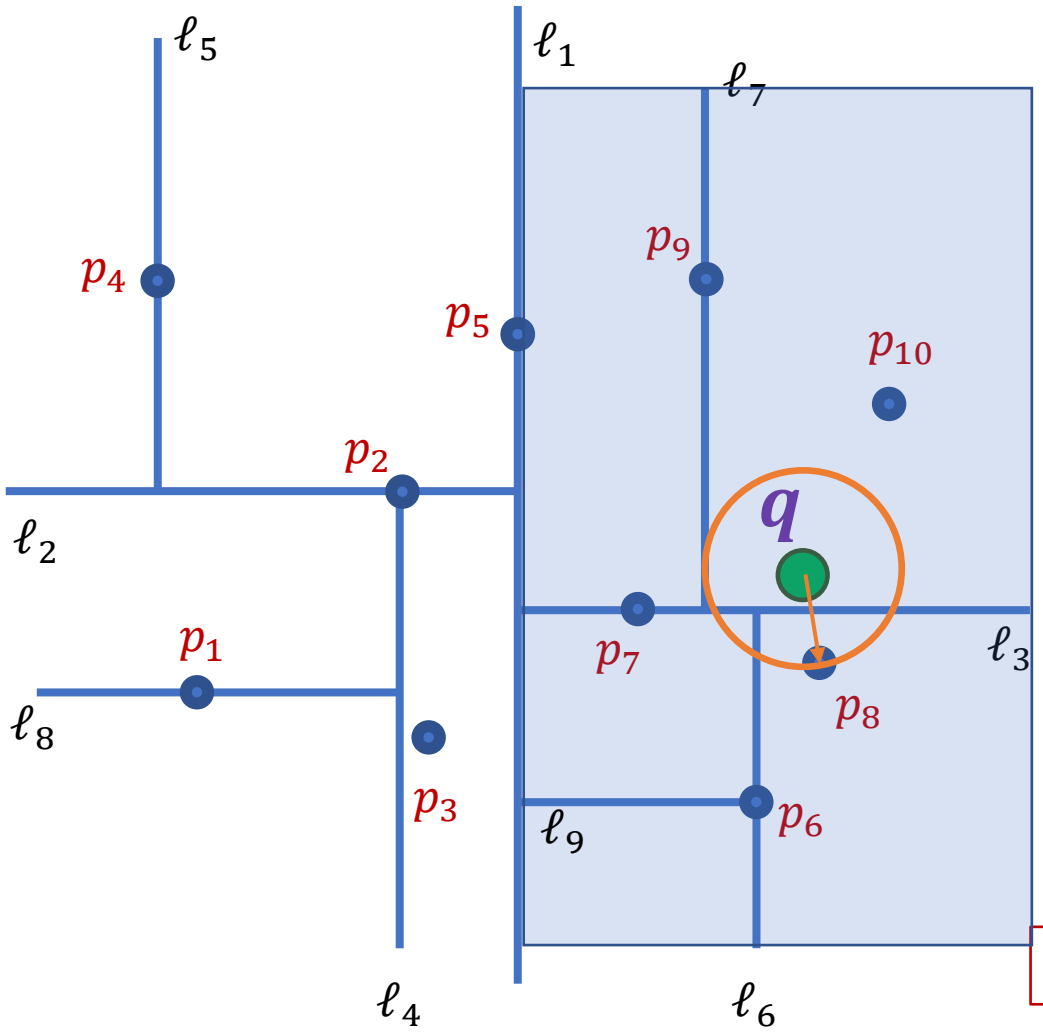


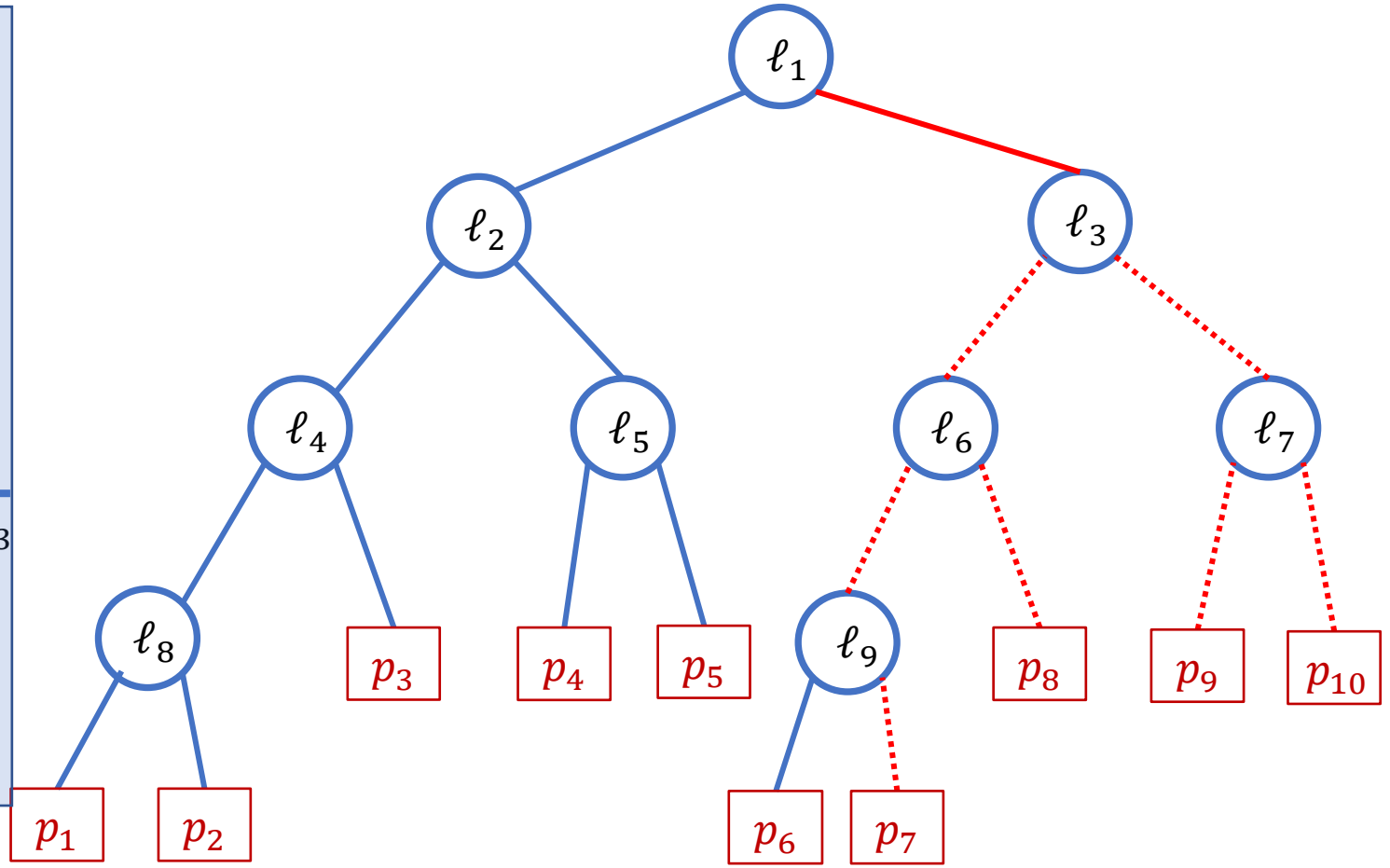
Current candidate: p_8





Current candidate: p_8





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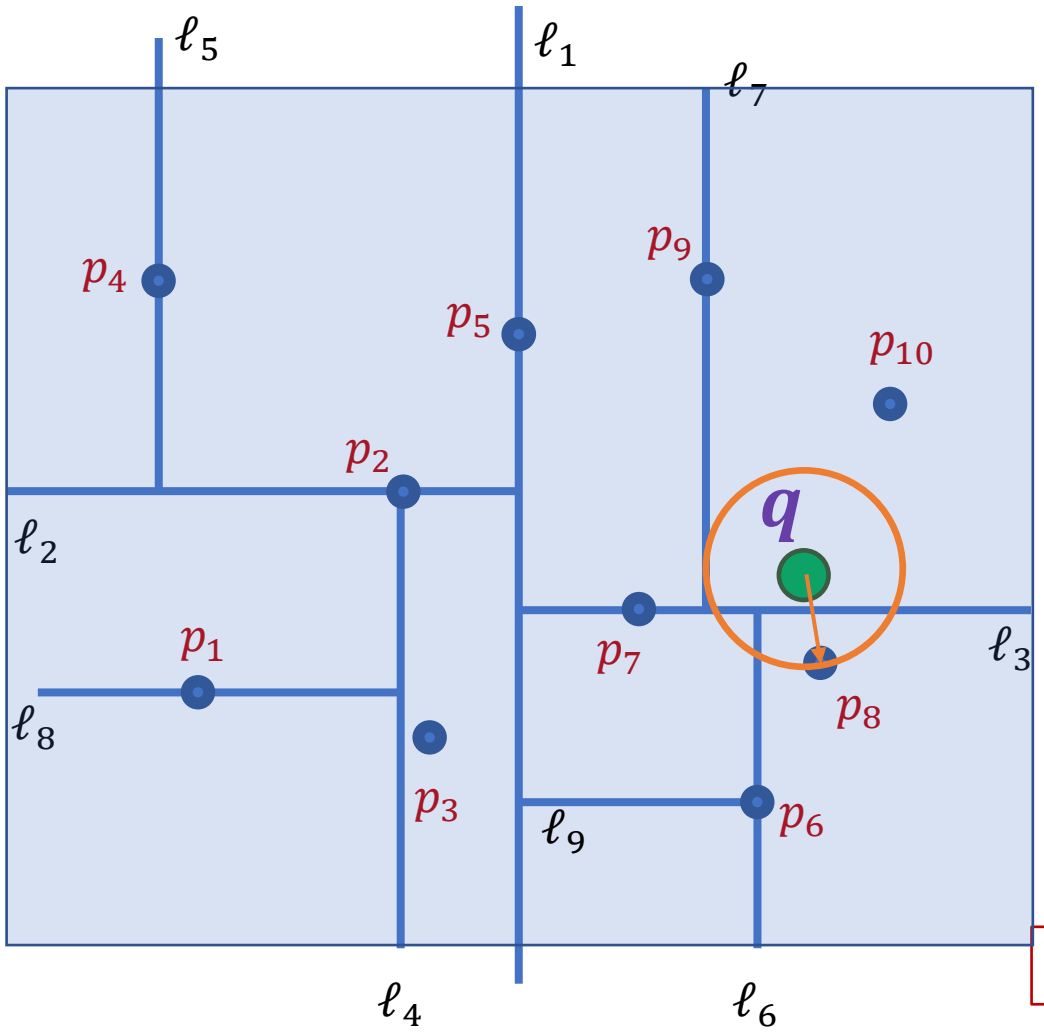
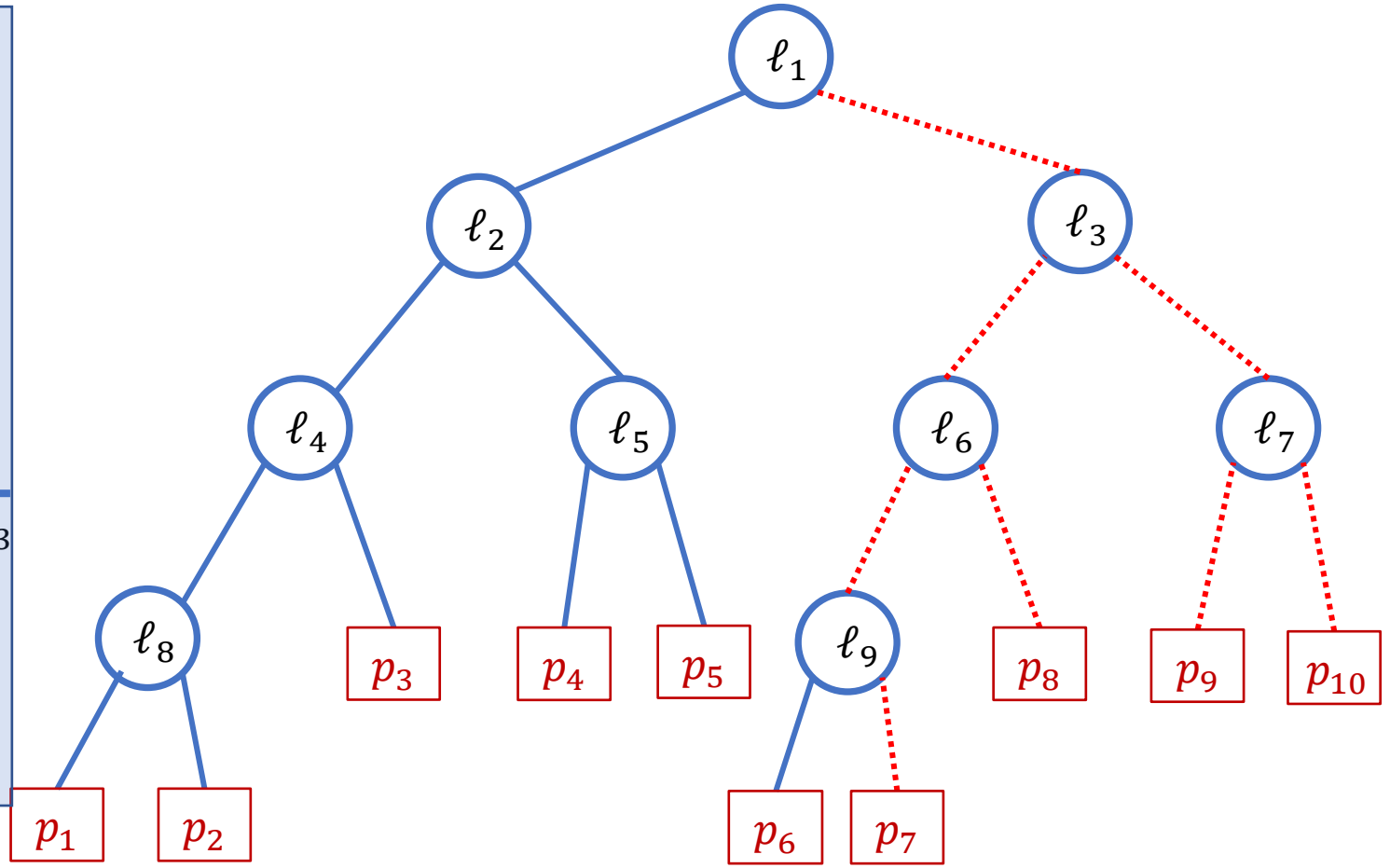
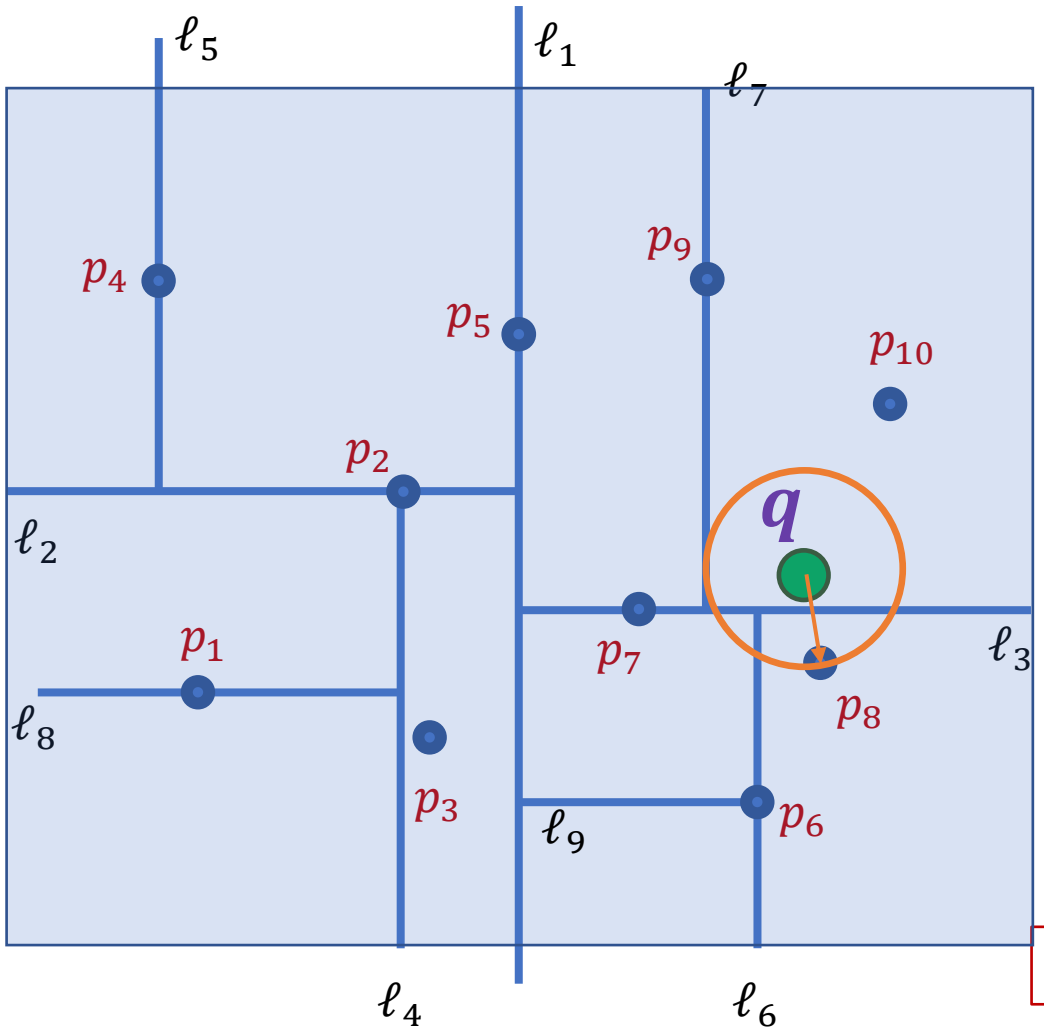
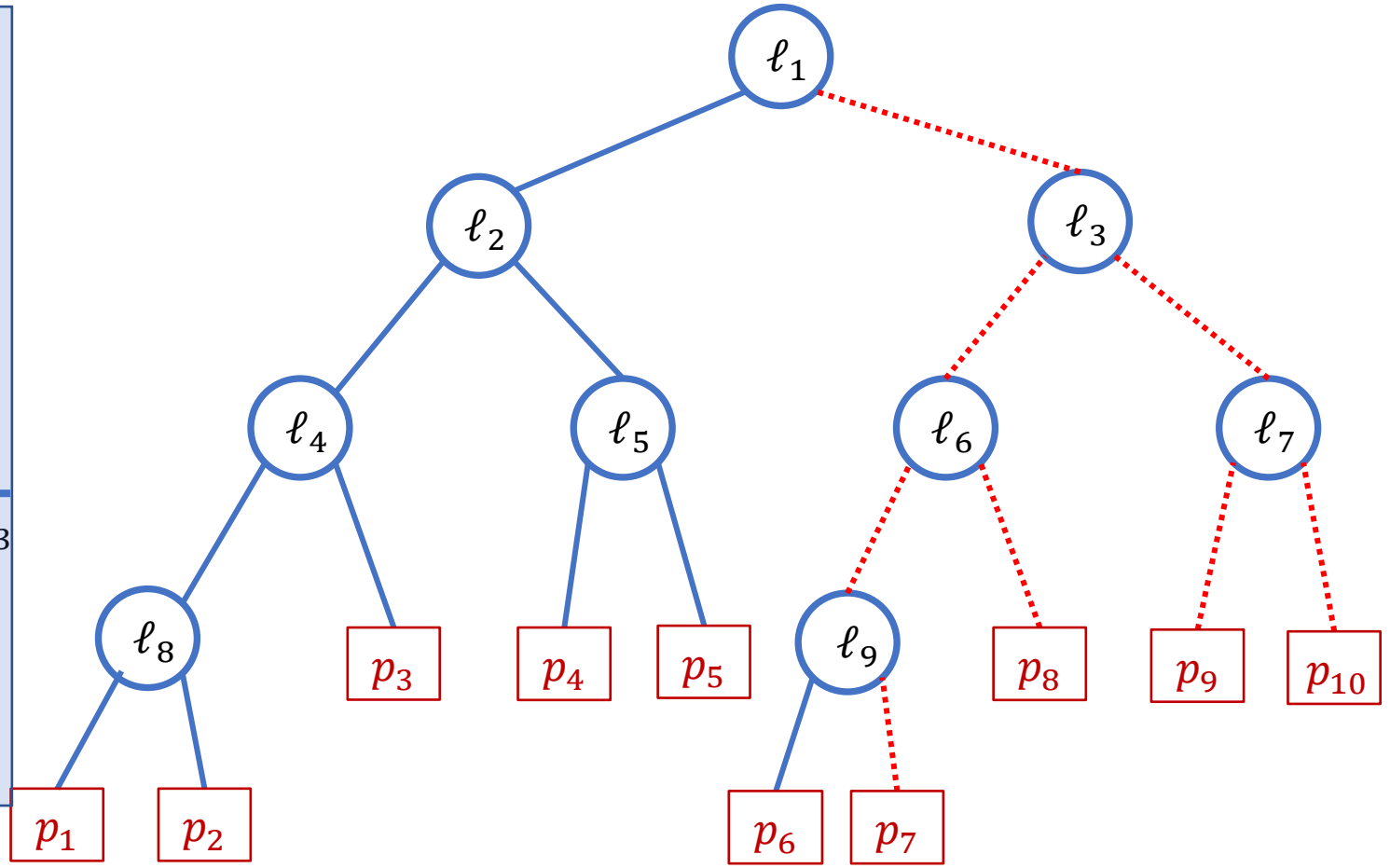




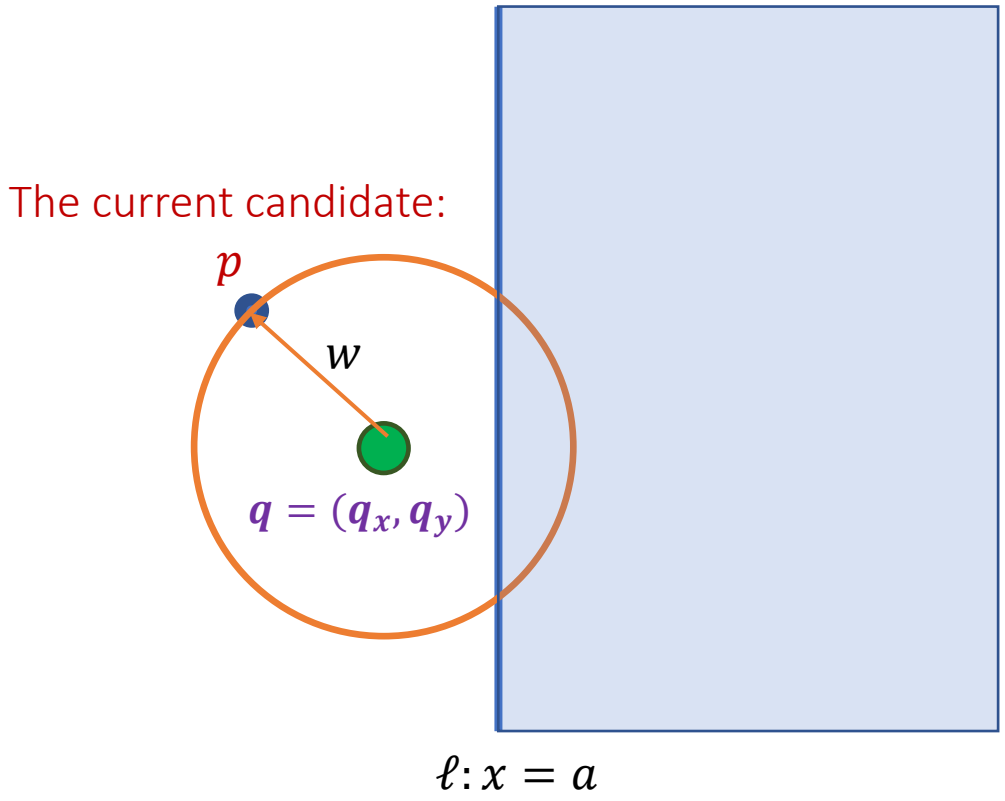
p_8 is the nearest neighbor





Testing for intersection

The box and the circle intersect if: $q_x + w > a$



Main is NNS(q,root,null,infinity)

Nearest Neighbor Search

```
NNS(q: point, n: node, p: point, w: distance) : point {
if n.left = null then {leaf case}
   if distance(q,n.point) < w then return n.point else return p;
else
   if w = infinity then
     if q(n.axis) ≤ n.value then
        p := NNS(q,n.left,p,w);
        w := distance(p,q);
        if q(n.axis) + w > n.value then p := NNS(q, n.right, p, w);
     else
        p := NNS(q,n.right,p,w);
        w := distance(p,q);
        if q(n.axis) - w ≤ n.value then p := NNS(q, n.left, p, w);
   else //w is finite//
      if q(n.axis) - w ≤ n.value then
      p := NNS(q, n.left, p, w);
      w := distance(p,q);
      if q(n.axis) + w > n.value then p := NNS(q, n.right, p, w);
   return p
}
```

Taken from https://courses.cs.washington.edu/courses/cse373/02au/lectures/lecture22l.pdf

Complexity

- For a “nicely spread” point set, NN-search on Kd-trees runs in $O(d \log n)$ average time
- In the worst case the query complexity is $O(dn)$
- Space complexity is $O(dn)$