Assignment no. 4

due: January 4th, 2021

Exercise 4.1 Prove that the number of inner nodes of the search structure $\mathcal{D}$ of algorithm TRAPEZOIDALMAP increases by $k_i - 1$ in iteration i, where k_i is the number of new trapezoids in the trapezoidal map $\mathcal{T}(S_i)$, and hence the number of new leaves of $\mathcal{D}$. (The exercise uses the notation of the book CGAA.)

Exercise 4.2 A polygon P is called star-shaped if a point p in the interior of P exists such that, for any other point q in P, the line segment pq lies in P.
(a) Given a simple polygon P with n vertices, describe a procedure running in expected $O(n)$ time, to determine whether P is star-shaped. Prove the correctness of the approach.
(b) Given a star-shaped polygon Q with n vertices, show that after expected $O(n)$ preprocessing time, one can determine whether a query point lies in Q in worst-case $O(\log n)$ time.

Exercise 4.3 Design an algorithm with running time $O(n \log n)$ for the following problem: Given a set P of n points, determine a value of $\varepsilon > 0$ such that the shear transformation $\Phi : (x, y) \to (x+\varepsilon y, y)$ (see Section 6.3 in CGAA) does not change the order, in x-direction, of points with unequal x-coordinates.

Exercise 4.4 Given a simple polygon P with n vertices and a query point q, here is an algorithm to determine whether q lies in P. Consider the ray $\rho := \{(q_x + \lambda, q_y) : \lambda > 0\}$ (this is the horizontal ray starting in q and going rightwards). Determine for every edge e of P whether it intersects ρ. If the number of intersecting edges is odd, then $q \in P$, otherwise $q \notin P$. Prove that this algorithm is correct, and explain how to deal with degenerate cases. (One degenerate case is when ρ intersects an endpoint of an edge. Are there other special cases?) What is the running time of the algorithm?

Exercise 4.5 Let L be a set of n lines in the plane. Give an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains all the vertices of the arrangement $\mathcal{A}(L)$ in its interior.

Exercise 4.6 Hopcroft's problem is to decide, given n lines and n points in the plane, whether any point is contained in any line. Give an $O(n^{3/2} \log n)$ time algorithm to solve Hopcroft's problem. Hint: Give an $O(n \log n)$ time algorithm to decide, given n lines and $\sqrt{n}$ points in the plane, whether any point is contained in any line.

Exercise 4.7 Let S be a set of n segments in the plane. A line ℓ that intersects all the segments of S is called a transversal or stabber for S.
(a) Give an $O(n^2)$ algorithm to decide if a stabber exists for S.
(b) Now assume that all the segments in S are vertical. Give a randomized algorithm with $O(n)$ expected running time that decides if a stabber exists for S. (CGAA Ex. 8.16)