
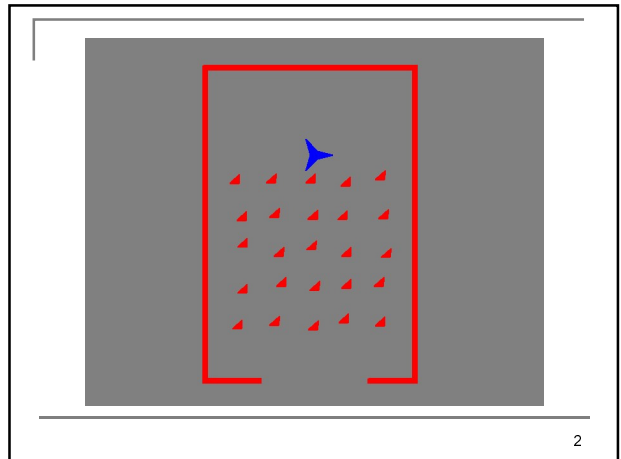


Motion Planning for Robots and other Creatures



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Overview

Part I: background and brief history
Part II: tools of the trade: Minkowski sums

הערות

- השקפים באנגלית, ההרצאה בעברית
- בציטוטים, שימו לב לשנה [Khatib 86]
- חלק ב', אם הזמן יתיר


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Overview

Part I: background and brief history
Part II: tools of the trade: Minkowski sums

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Robotics



RAS field of interest (ICRA, Rome, April 2007):

Robotics focuses on sensor and actuator systems that operate autonomously or semi-autonomously (in cooperation with humans) in **unpredictable** environments. Robot systems emphasize intelligence and adaptability, may be networked, and are being developed for many applications such as service and personal assistants; surgery and rehabilitation; haptics; space, underwater, and remote exploration and teleoperation; education, entertainment; search and rescue; defense; agriculture; and intelligent vehicles.



Motion planning: the basic problem

Let B be a system (the robot) with k degrees of freedom moving in a known environment cluttered with obstacles. Given free start and goal placements for B decide whether there is a collision free motion for B from start to goal and if so plan such a motion.

Two key terms: (i) degrees of freedom (dofs) and (ii) configuration space

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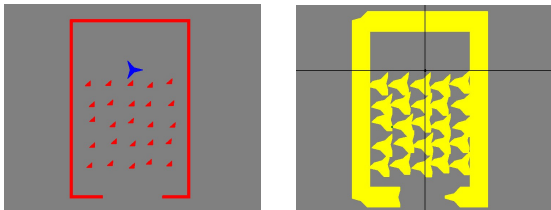
What is the number of DoF's?

- a polygon robot translating in the plane
- a polygon robot translating and rotating
- a spherical robot moving in space
- a spatial robot translating and rotating
- industrial robot arms



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Configuration space



[Lozano-Perez, late 70s]

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Complete solutions

- the problem is hard when the number of degrees of freedom (# dof) is part of the input [Reif 79], [Hopcroft et al. 84], ...
- the Piano movers series [Schwartz-Sharir 83], **cell decomposition**: a doubly-exponential solution
- **roadmap** [Canny 87]: a singly exponential solution

Algorithms in Real Algebraic Geometry [Basu, Pollack, Roy 03,06]

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dof



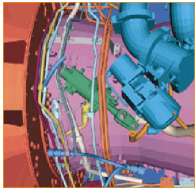
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Near-optimal solutions for small k

- assuming the robot is fixed, the problem size is the complexity of the obstacles
- k=2, **near-linear**
polygon robot, translation [Edelsbrunner-Guibas-Sharir]
general [Guibas-Sharir-Sifrony 89]
- k=3, **near-quadratic**
polyhedron robot trans [Aronov-Sharir 88]
polygon robot, trans+rot [Halperin-Sharir 93]
general [Halperin-Sharir 94, Schwarzkopf-Sharir 96]
- k>=4
rod in space, near-optimal [Kotun 05]
otherwise, efficient solutions, suboptimal

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Meanwhile in robotics

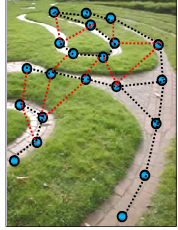
- potential field methods [Khatib 86]
 - attractive potential (goal), repulsive potential (obstacles)
- random path planner (RPP) [Barraquand, Latombe 90]
 

[Chang, Li 95]

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Meanwhile in robotics

- potential field methods [Khatib 86]
 - attractive potential (goal), repulsive potential (obstacles)
- random path planner (RPP) [Barraquand, Latombe 90]
- and then, around 1995
 - PRM (Probabilistic RoadMaps) [Kavraki, Svestka, Latombe, Overmars]
 - many variants followed, e.g. RRT (Rapidly Exploring Random trees) [LaValle, Kuffner 99]



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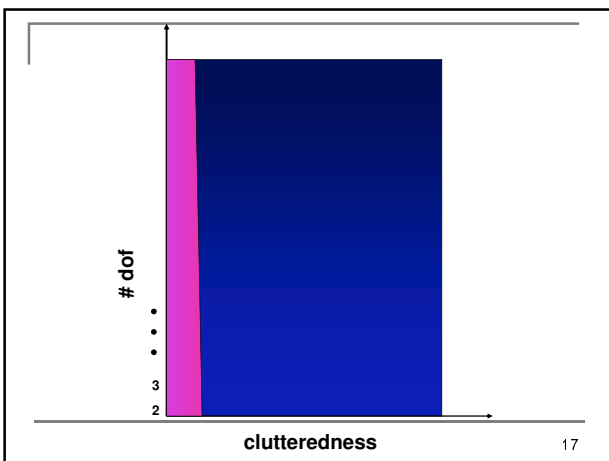
Sampling-based motion planners

- easy to implement, provided you have a good static collision detector [Lin, Manocha et al; survey, Hdbk of DCG 04]
 
- extended the applicability of motion planning: animation, docking motions, virtual prototyping, more
 

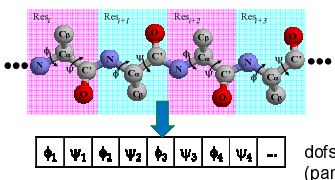
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Is motion planning solved?

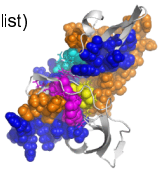
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Proteins as robots

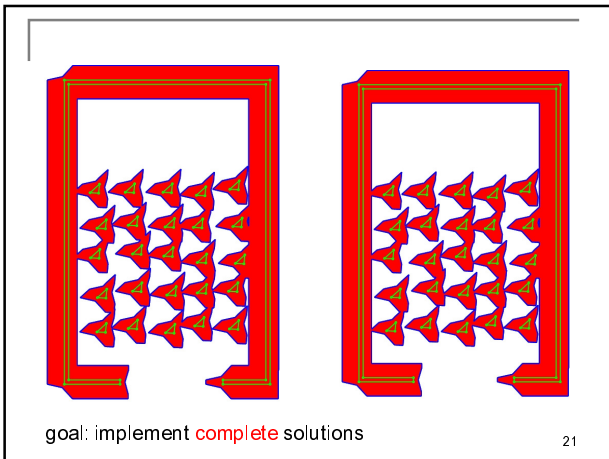
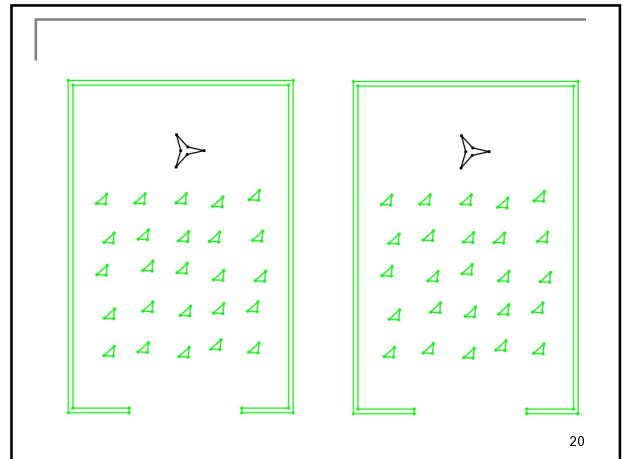
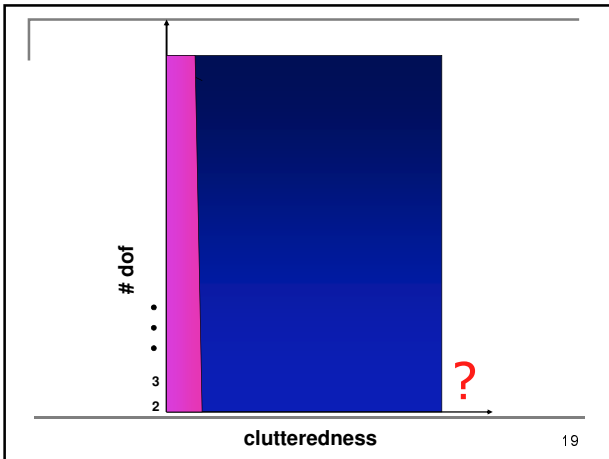


- Robot \rightarrow peptide chain
- Obstacles \rightarrow steric clashes between atoms
- Collision-free path \rightarrow a low-energy motion pathway, free of steric clashes




dofs (partial list)

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Effective exact planners

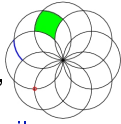


- goal: implement **complete** solutions
- problems:
 - degeneracies
 - algebraic operations
 - arithmetic precision
 - misleading performance measures: asymptotic bounds, 'unit' cost

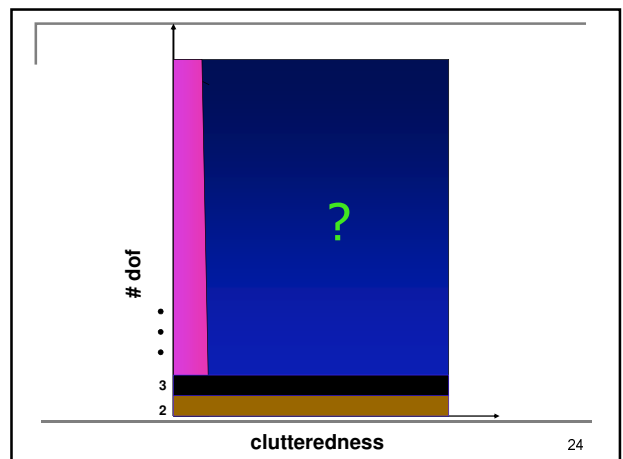
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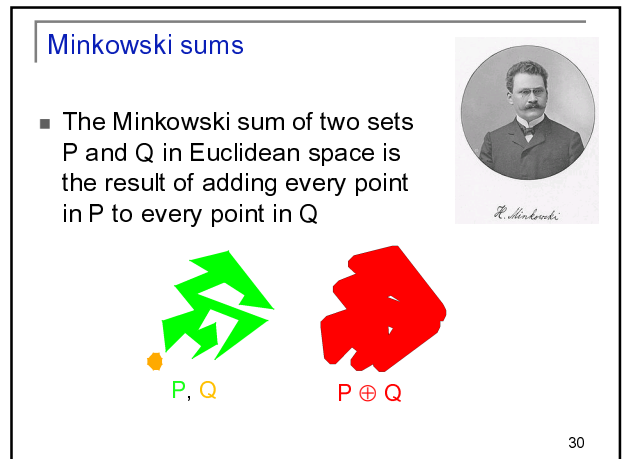
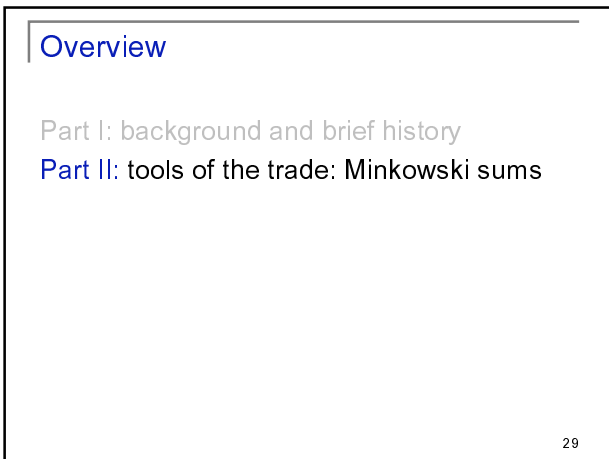
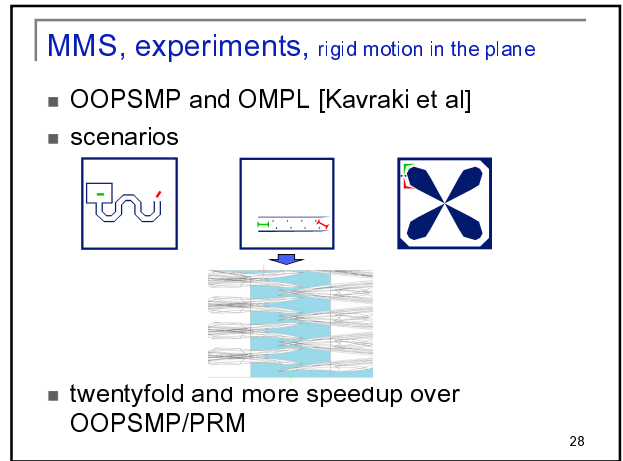
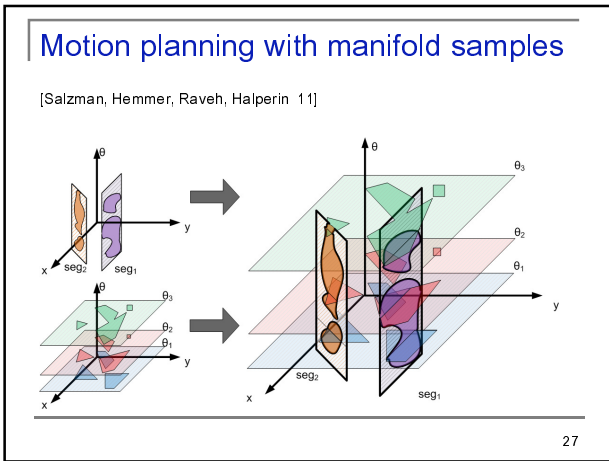
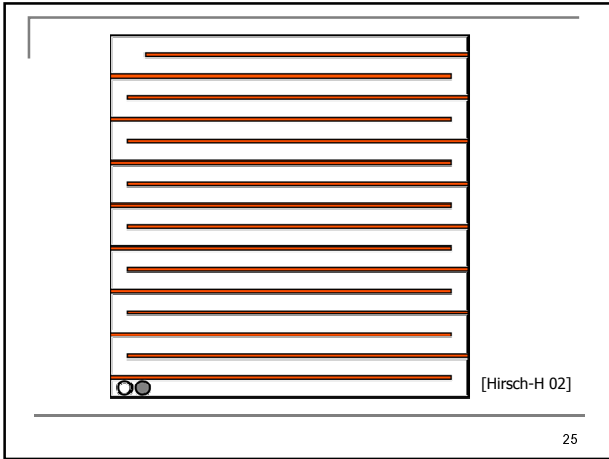


- computational geometry algorithms library, www.cgal.org
- emphasis on robustness issues
- Tel Aviv: maps and arrangements, useful tools for representing configuration spaces, acg.cs.tau.ac.il



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Minkowski sums and translational motion

R - a polygonal object that moves by translation
 P - a set of polygonal obstacles

reference point

When translating, R intersects P iff $\text{ref}(R)$ is inside $P \oplus -R$

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Minkowski sums and much more

- minimum separation distance
- placement
- tolerancing, offsetting
- nesting
- cartographic generalization

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Non trivial bound upper bound: convex and non-convex

input	sum complexity
P is convex Q is convex	$\Theta(m+n)$
P is convex Q is general	$\Theta(m \cdot n)$ [KLPS]
P is general Q is general	$\Theta(m^2 \cdot n^2)$

P with m vertices, Q with n vertices

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video

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Studying motion planning further

- the expansion of robotics, new designs and demanding requirements
- difficult variants: non-holonomic, kino-dynamic, deformable objects, moving obstacles
- a mixture of algorithms and data structures, combinatorics, algebra, topology, and more
- good solutions to motion-planning problem (in robotics and in computational geometry) traditionally had repercussions far beyond robot motion
- exciting challenges in algorithms and engineering

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THE END

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