Arrangements of Geodesic Arcs on the Sphere

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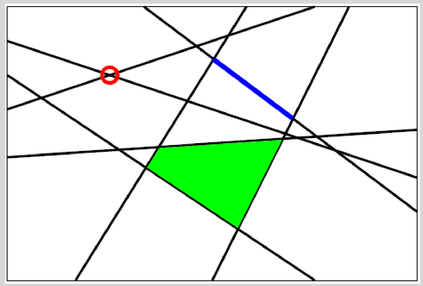
Abstract

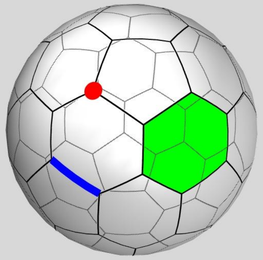
The Arrangement_2 package of CGAL (Computational Geometry Algorithms Library) develop at Tel Aviv university has recently been extended to support arrangements of curves embedded on two-dimensional parametric surfaces. The Arrangement_2 package can be used to construct and maintain arrangements induced by arcs of great circles embedded on the sphere in an exact yet efficient manner. An application of this new development is the ability to compute various types of Voronoi diagrams on the sphere. The resulting diagrams are represented as arrangements and can be passed as input to consecutive operations supported by the Arrangement_2 package.

Introduction

Arrangements

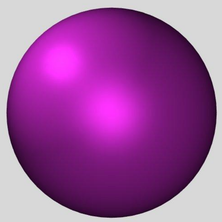
Given a collection of curves, their **arrangement** is the partition of the ambient space into cells



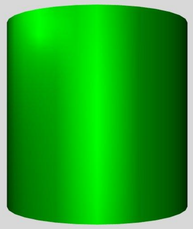


Bivariate Parametric Surfaces

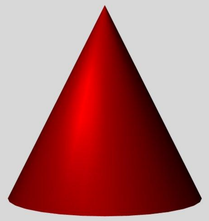
A **parametric surface** is a surface which is defined by parametric equations involving two parameters



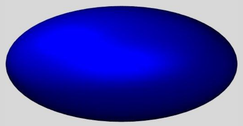
Sphere



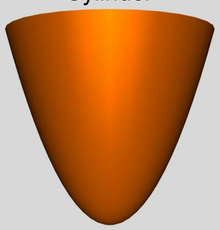
Cylinder



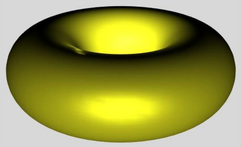
Cone



Ellipsoid



Paraboloid



Torus

The general framework was developed together with Eric Berberich, Ron Wein and Kurt Mehlhorn.

The Parameter Space

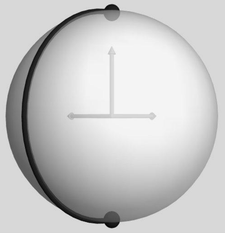
The Parameter Space

Cylinder

$$S(u,v) \in \begin{cases} r\cos u \\ r\sin u \\ v \end{cases} \qquad u \in [-\pi,\pi], v \in R$$

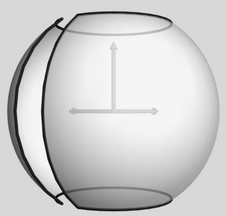
Sphere

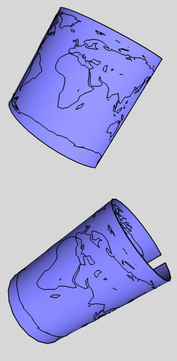
$$S(u,v) = \begin{cases} r\cos u\cos v \\ r\sin u\cos v \\ r\sin v \end{cases} \qquad u \in [-\pi,\pi], v \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$



The Modified Parameter Space

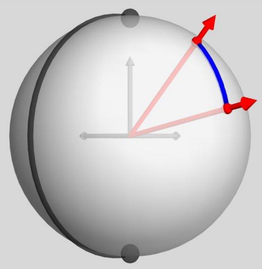
Identification curves, contraction points, and points at infinity are removed from the parameter space to yield the modified parameter space. The heart of the work is in reducing the original problem on a surface into the well-studied case of planar arrangements while efficiently accounting for the topological modification.





Geometric Representation of Geodesic Arcs

- A Point is represented by a direction in space
- A geodesic arc is represented by the source and target endpoints and the normal of the plane that contains the two endpoint directions and the origin (determines which one of the two possible geodesic arcs is considered)

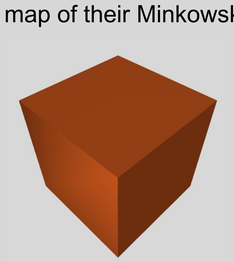


- This representation enables an exact yet efficient implementation of all geometric operations using only exact rational arithmetic
- Normalizing directions and plane normals is completely avoided

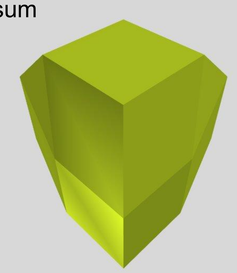
Applications

Minkowski Sums of Polytopes

The overlay of the Gaussian maps of two polytopes is the Gaussian map of their Minkowski sum

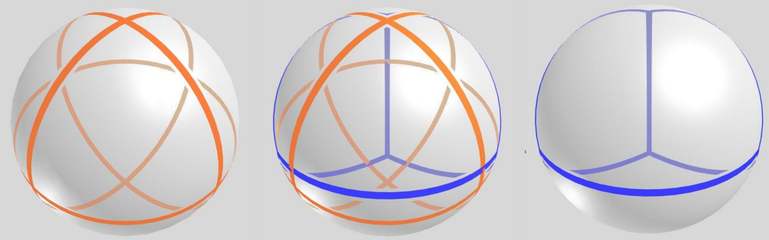


Cube



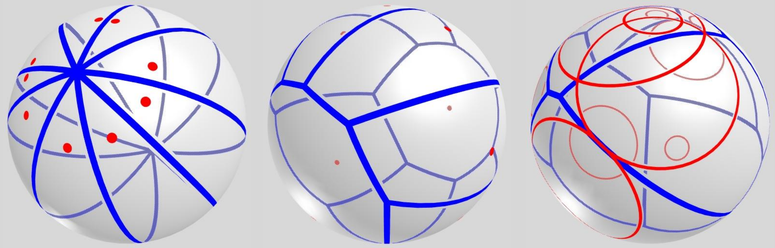
Cube ⊕ Tetrahedron

Tetrahedron



Voronoi Diagrams on the Sphere

Voronoi diagrams are a subdivion of the ambient space into regions that have the same closest object. Using geodesic arcs we can compute Voronoi diagrams on the sphere. On the right-hand side is an example of the Power (or Laguerre) diagram on the sphere.



Operations on Arrangements

The **overlay** of:

- an arrangement on the sphere induced by the continents and some of the islands on earth rendered in blue
- The Voronoi diagram the cities that hosts the institutions that participate in the ACS (Algorithms for Complex Shapes) project rendered in red

More information can be found in: http://cgal.cs.tau.ac.il/projects

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